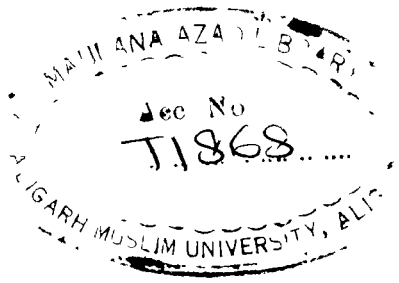
A Study of Short Range Nucleon-Nucleon Correlations in Light Nuclei

(SUMMARY)



By

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M.Sc. (Alig.)

THESIS SUBMITTED TO THE ALIGARH MUSLIM UNIVERSITY, ALIGARH IN PARTIAL FULFILMENT FOR THE AWARD OF PH. D. DEGREE IN

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SUMMARY

In the present study an attempt has been made to extract information on short range correlations in light nuclei through elastic electron scattering. In this regard, there have been controversial discussions in the literature whether introduction of short range correlations over the single particle orbitals leads to a meaningful description of processes involving the so called 'one body operators', e.g. elastic electron scattering experiments[1-5]. It seems to be generally believed that given a set of single particle orbitals and a Jastrow correlation factor, the resulting one body density can be reproduced with a different set of single particle orbitals. This has led some to believe, e.g. see ref. 4), that elastic electron scattering is sensitive only to the single particle aspect of the total nuclear wavefunction. On the other hand, some argue, e.g. see ref. 5), that the findings of Ripka et.al.[1] make it rather arbitrary to introduce a limit in the one body density below which the single particle properties prevail and above which the correlations are important, thus, elastic electron scattering is neither sensitive to the single particle nor to the correlation aspects of the total nuclear wavefunction. In the present study we have tried to resolve the above controversy. By making a particle-hole expansion of the Jastrow wavefunction it is shown that the correlations manifest because of their

interplay with the centre of mass correlation even for processes which involve one body intrinsic operators. It is argued that for light nuclei at large momentum transfers the particle-hole expansion, though mathematically exact, may lead to wrong conclusions unless the effect of the centre of mass motion is taken into account from the very beginning. This is found to be the reason which led the earlier authors[1,2] to conclude that elastic electron scattering is fundamentally incapable of distinguishing between the shell model and the correlation aspects of the wavefunction. Thus, our results and conclusions are in direct contradiction with the findings of Ripka et.al.[1] and Pink et.al.[2] who considered the removal of centre of mass motion as merely a correction to be applied on the charge form factor calculations.

In earlier analyses[6], where attempts were made to extract information on short range correlations through elastic electron scattering, it was found that several correlation factors give equally good fits to the data. This was thought reasonable as the elastic electron scattering involves a one body operator. The above finding was also in conformity with the conclusions of ref. 1,2). On the contrary, we have argued and shown that the above insensitivity of the data towards the Jastrow correlation factor is a consequence of availability of the data only in a finite range of momentum transfer and is not related to the one body character of the scattering process. Thus a definite determination of short range correlations is not

possible by analysing the data in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}. The above indeterminacy arises out of the indeterminableness of the one body density due to the availability of the data upto some finite maximum momentum transfer q_{max}. Nevertheless, the correlation effects are shown to be independent of the single particle orbitals employed and it is argued that the wavefunctions obtained in the present study may be used only upto $\sim q_{max}/2$ in processes which involve two body intrinsic operators. In this regard, the recent ^{4}He data of Arnold et.al.[7] upto 8 fm^{-1} would be useful. It would certainly give more detailed and reliable information on short range correlations.

The thesis is divided into three chapters. In chapter I the Jastrow wavefunction is defined and the expressions for the charge form factor and other related quantities in the linked cluster formalism without the centre of mass effect are given. The work of Ripka et.al. which claims the inadequacy of elastic electron scattering to furnish any information on short range correlations is described. We then emphasise the importance of the centre of mass effect and show that the conclusions of Ripka et.al. and Fink et.al. are not correct, particularly for light nuclei. It is argued that an unambiguous determination of correlations is not possible by analysing the data in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}. In chapter II, linked cluster expansion and independent pair approximation with centre of mass effects included are developed for the charge form factor.

The Jastrow wavefunction is broken into particle-hole excitations and contributions to charge form factor due to one particle - one hole (1p-1h) and two particle - two hole (2p-2h) excitations are separated out. This is carried out exactly for ^{4}He and an approximate procedure is developed for other nuclei. A highly flexible correlation factor is proposed which is found useful in showing the independence of the correlation effects with respect to the single particle orbitals. A method is described for calculating the matrix elements involving the proposed correlation factor. Finally, the matrix elements involved in the charge form factor calculations are explicitly calculated. In chapter III, numerical calculations for ^{4}He, ^{6}Li and ^{16}O are carried out. Results and their implications are discussed. In the end important calculations of the present study are given.

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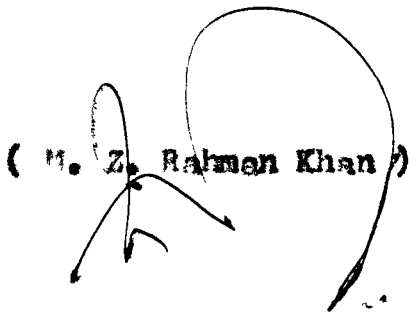
Certified that the work presented in this thesis is the original work of Mr. Q. N. Usmani, carried out under my supervision.



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C O N T E N T S

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CHAPTER - I

1. INTRODUCTION.

The study of short range correlations in nuclei is important mainly from two angles. Firstly, it might yield some information about two nucleon force at short distances. Secondly, the calculations employing the realistic interactions give wavefunctions which do not generally account for experiments involving high momentum transfers[1]. It would be useful to obtain wavefunctions from some set of experiments in a reliable way and then use them to explain other experimental data. Consequently, there has been wide interest in recent years for the experimental search and theoretical description of short range correlations in nuclei. We mention below the important processes where studies have been made about short range correlations :

(i) One nucleon knock-out reactions like (e,e'p) and (p,2p) [refs.2,3].

(ii) Elastic scattering of high-energy projectiles [refs.4-11].

(iii) Absorption processes with either one or two nucleons emitted (γ, N), (π,N), (γ,2N) and (π,2N) [refs. 12-14].

(iv) Deuteron knock-out, (e,e'd) and (p,p'd)[refs.15-18].

In the present study, we attempt to determine nuclear wavefunctions and in particular examine their behaviour at short internucleon distances from elastic electron scattering. In this connection there have been controversial discussions in the literature[19-23] as to whether the introduction of short range correlations over the single particle orbitals leads to a meaningful description of experiments involving the so called "one body operators". In the present study we have tried to resolve the above controversy. This has been done by considering the much discussed elastic electron scattering experiments which involve the one body operator.

The high energy electron scattering experiments have been extensively used to elicit information on the structure of nuclei, in particular the charge density distribution[24-29]. These experiments have provided a measurement of fine nuclear structure effects such as the investigation in detail of the wavefunction of ground and excited states, the measurement of nuclear separation energies and the production of baryon resonance in nuclei. As indicated earlier, an interesting aspect of these experiments, about which we shall be concerned in the present study, has been the investigation of the origin of high momentum components in the nuclear wavefunction. This problem attracted much attention in late sixties and early seventies when high energy data on elastic electron scattering appeared. With the observation of the diffraction

structure in the elastic scattering of electrons [25-29] from ^{3}He, ^{4}He, ^{6}Li, ^{16}O and ^{40}Ca in the high momentum transfer region, $2.5 \leq q \leq 4.5$ fm^{-1}, one finds a failure of the pure oscillator model which could explain the data at low momentum transfers. With this fact in mind and the realisation that the high momentum components in the nuclear wavefunction may arise from the strong short range repulsion and the attractive part just outside the repulsive core in the nucleon-nucleon potential[30], authors like F.C.Khanna[7], C.Ciofi Degli Atti[4,31] and others [6,7] introduced correlations, in a phenomenological way, over the simple oscillator Slater determinant as suggested by Jastrow[32]. They found that charge from factor data could be well explained with a Jastrow correlation factor which takes into account the strong repulsion at small relative internucleon distances in the nucleon-nucleon force. The main conclusions of these authors are summarised below :

(1) The deviations from the independent particle model can be systematically removed by Jastrow correlations to account for the charge form factor data.

(2) Correlation effects seem to be independent of the mass number and the single particle orbitals.

(3) Various Jastrow correlation factors give equally good fit to the data. This was thought reasonable since the form factor

is the expectation value of a one body operator and is connected with correlations only through the high momentum components in nuclear wavefunction.

However, all the attempts to explain the charge form factor data in the high momentum transfer region with the Jastrow correlation approach were severly criticized, in particular, by Fink, Hebach and Kummel[21] and Ripka, Gillespie and Gaudin[23]. It was shown that the connection between correlations and high momentum components is not reciprocal in the sense that correlations give rise to high momentum components, but those components are not in themselves a proof of the existence of correlations. They[23] demonstrated that given a set of single particle orbitals and a Jastrow correlation factor, the resulting one body density can be reproduced with a different set of single particle orbitals. It was argued that if Pauli operator is included in the correlation factor of the Jastrow theory the correlation contribution disappears in the first order but manifests itself through the modified single particle orbitals. This implied that elastic electron scattering, instead of giving information about short range correlations, is sensitive to the single particle aspect of the nuclear wavefunction . At the same time various charge form factor calculations appeared with " single particle" wavefunction which demonstrated that elastic electron scattering could be explained, at least, qualitatively, without introducing

correlations[33]. Therefore, the criticism of Fink et.al.[21] and Ripka et.al.[23] coupled with the fact that the elastic electron scattering data could, at least, be qualitatively explained with single particle wavefunctions gave a completely different meaning to all the above mentioned Jastrow theoretic calculations. It created the impression that the separation of one body density into a correlated and an uncorrelated part can not be made experimentally from a measurement of density alone and any attempt to do so is fundamentally not correct.

However, the validity of the above mentioned conclusions is of doubtful nature because the important effect of the centre of mass motion has been either not taken into account or not considered properly. The Slater determinant is a function of A non-intrinsic coordinates, where A is the number of nucleons. The calculation of intrinsic matrix elements with non-intrinsic coordinates is a very difficult task; only various approximate prescriptions can be given to remove the centre of mass coordinates, and the extent to which they lead to different results is a measure of the importance of the spurious centre of mass components in nuclear wavefunction. It has been demonstrated by Friar[34] and Degli Atti et.al.[35] that all of the single particle-type successful interpretation of charge form factor data, particularly of ^{4}He, is due to the spuriousity effects present which are large at high momentum

transfers, rather than it has relevance to the parameters of the single particle model.

In the present work we intend to show that the centre of mass effect plays a crucial role in the interpretation of the electron scattering data and that the conclusions of Rinker et.al[23] and Fink et.al.[21] are only correct for relatively heavy nuclei where the omission of the centre of mass effect may not be inappropriate in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}. We shall demonstrate that for light nuclei the elastic electron scattering is a valuable tool in investigating the correlation structure of nuclei and that the relationship between the correlation and the high momentum components in the nuclear wavefunction is one of reciprocity. We shall obtain information about short range correlations, which we define as consisting of two particle-two hole excitations in the model single particle wavefunction, thus showing that elastic electron scattering is fundamentally capable of distinguishing between the simple shell model and the correlation aspect of the total nuclear wavefunction. Therefore, given a one body density one may find a two body density etc.. This result is of great utility. For instance, one of the basic problems of hadron-nucleus collisions in the intermediate energy region is whether the discrepancy between the theory and experiment is due to nuclear structure (short range correlations and two body density) effects or due to inadequacy of reaction formalism, uncertainty

associated with nucleon-nucleon amplitude and off-shell effects. This can be decided once the short range correlations and the two body density is determined from elastic electron scattering. A similar situation arises in ion-ion collisions at intermediate energies. There also some information about reaction mechanism can be obtained if the nuclear structure effects are eliminated through elastic electron scattering. Several other examples may also be cited.

There are other ways also where information about short range correlations may become possible in experiments involving directly the two body density. For instance, one may hope[10-14 36,37] that the pion production and absorption experiments will tell us something about short range correlations. The argument goes like this. The process of absorption or emission of a low energy pion involves a large change in the energy of the system but the change in momentum is negligible. This change in energy in turn implies a large change in momentum of relative motion of two nucleons which is not available from their motion in the average field but is provided by the short range correlations. This expectation has been found to be correct experimentally[37]. Absorption of pion in pionic atom is almost entirely due to absorption by a correlated pair. One sees two nucleons coming out. Unfortunately, the reaction involves three body final states and this makes extraction of information on

correlations quite difficult. On the other hand, no such problem would arise in the study of short range correlations through elastic electron scattering.

In earlier analyses[4,7,31], where attempts have been made to extract information about short range correlations, it has been found that several correlation factors give equally good fit to the data. This finding has been attributed to the fact that since elastic electron scattering involves a one body operator it measures the short range correlation indirectly. We, on the contrary, show that this insensitivity of the elastic electron scattering towards the Jastrow correlation factor may be a consequence of the availability of data only upto a certain finite maximum momentum transfer, q_{max}, and is not related to the so called "one body" character of the scattering process. We show that the use of model correlation factors (e.g.Gaussian or spherical Bessel function type) may be quite misleading. We, therefore, propose a highly flexible correlation factor of arbitrary range and shape by expanding it in terms of its moments. This expansion is motivated by the fact that the correlation factor is of short range and appears as an effective potential in the various matrix elements involved in the calculation. Such a parametrisation of the correlation factor is similar to the Skyrme parametrisation of the two nucleon force.

In the present work we have studied three nuclei, namely ^{4}He, ^{6}Li, and ^{16}O. The nucleus ^{40}Ca is left out, partly because the adequacy of Born approximation for such a heavy nucleus is doubtful and partly because, as it would become apparent from our analysis of the three above mentioned light nuclei, the short range correlation effects in this nucleus would not be important in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}. We have also not studied ^{12}C nucleus. The reason for this shall be given in the last chapter.

We have completely neglected the relativistic effects and the contribution due to meson exchange currents. It may be possible to incorporate them. However, as we shall point out, their inclusion at the present stage is not warranted unless we have data upto sufficiently high momentum transfers.

The organization of the thesis is as follows :

In the following sections of this chapter the Jastrow wave-function is defined and the expressions for the charge form factor and other related quantities in the linked cluster formalism without the centre of mass effect are given. The work of Ripka et.al [3], which claims the inadequacy of elastic electron scattering to furnish any information on short range correlations is described. We then emphasize the importance of the centre of mass effect and show that the conclusions of Ripka et.al.[3] and Wilk et.al.[1] are not correct, particularly

for light nuclei. We also give a critique of the earlier attempts [4,7,31] to interpret the elastic electron scattering data with Jastrow correlations. In chapter II, we obtain expressions for the charge form factor in linked cluster expansion and independent pair approximation with centre of mass effects included. The Jastrow wavefunction is broken into particle-hole excitations and the contributions to charge form factor due to one particle-one hole (1p-1h) and two particle-two hole (2p-2h) excitations are separated out. This is carried out exactly for ^{4}He and an approximate procedure is developed for other nuclei. We then propose a highly flexible correlation factor which we find necessary to obtain single particle model independent information on short range correlations. We then describe a method for calculating the matrix elements involving our proposed correlation factor. Finally, the matrix elements involved in charge form factor calculations are explicitly calculated. In chapter III, numerical calculations for ^{4}He, ^{6}Li and ^{16}O are carried out. Results and their implications are discussed. Finally, important conclusions of the present study are described.

2. DEFINITION OF JASTROW WAVEFUNCTION AND THE CHARGE FORM FACTOR IN JASTROW THEORY :

The use of Jastrow wavefunction in nuclei has developed greatly after the pioneering work of Iwamoto and Yamada[33]. These wavefunctions are used for strongly interacting systems

where ordinary p rturbation theory becomes inadequate. Although, the Jastrow wavefunctions are simple in structure, it is very difficult to calculate exactly the expectation value of one and two body operators with them. Various cluster expansions have been proposed to simplify matter. Most of the charge form factor calculations have been performed employing either linked cluster expansion or independent pair approximation. We shall give here only relevent expressions and formulae. Details may be found in ref.23).

A Slater determinant or a sup rposition of a finite number of Slater determinants is taken as the zero order approximation. The 'complete' wavefunction for a system of A nucleons is given by

$$\Psi(X_1, X_2, \ldots X_A) = \frac{1}{\sqrt{N}} \prod_{1 \le i < j \le A} f(r_{ij}) \phi(X_1, X_2, \ldots X_A) \quad (1.2.1)$$

N is a normalization constant such that

$$\int |\Psi(X_1, \ldots X_A)|^2 dX_1 \ldots dX_A = 1 .$$

The symbol X stands for space, spin and isospin coordinates and dX stands for spatial integration plus sum over spins and isospins. $\phi(X_1, \ldots X_A)$ is a Slater determinant made up of A single particle wavefunctions $|\phi_\alpha(X)|$, the ensemble of which is called the Fermi sea F

$$\phi(X_1, \ldots, X_A) = \det |\phi_\alpha(X_i)|, \alpha \in F, i \in [1, A] \quad \ldots \quad (1.2.2)$$

and

$$f(r_{ij}) = f(|\vec{r}_i - \vec{r}_j|) \qquad \ldots \qquad (1.2.3)$$

is a two body correlation factor which accounts for the deviation of the total wavefunction from the single particle behaviour. We shall also be using auxiliary correlation factors, $b(r_{ij})$ and $h(r_{ij})$, which are related to $f(r_{ij})$ through the relations,

$$b(r_{ij}) = |f(r_{ij})|^2 - 1, \qquad \ldots \qquad (1.2.4a)$$

$$h(r_{ij}) = f(r_{ij}) - 1 . \qquad \ldots \qquad (1.2.4b)$$

We now define the one body correlation function

$$G(X_1, X_1') = A \int \Psi^*(X_1, X_2, \ldots, X_A)\, \Psi(X_1', X_2, \ldots X_A) dX_2 \ldots dX_A . \qquad (1.2.5)$$

The expectation value of one body operator $\theta_1(X)$ is then given by

$$\langle \theta_1 \rangle = \int |\theta_1(X_1) G(X_1, X_1')|_{X_1 = X_1'} dX_1 . \qquad \ldots \qquad (1.2.6)$$

The one body density $\bar{\rho}(X)$ is given in terms of the one body correlation function $G(X,X')$ by

$$\bar{\rho}(X) = G(X,X) . \qquad \ldots \qquad (1.2.7)$$

For a spin saturated nucleus, the density (1.2.7) is independent of the spin and isospin coordinates. If we define

$$\rho(\vec{r}) = \sum_{\alpha \in F}' |\phi_\alpha(\vec{r})|^2$$

$$\rho(\vec{r},\vec{r}') = \sum_{\alpha\epsilon F}{}' \phi_\alpha(\vec{r})\,\phi_\alpha(\vec{r}') \quad ,$$

where the sum Σ' are over the spatial wavefunction of the occupied orbits only (no spin or isospin sum), the correlated density $\bar{\rho}(\vec{r})$ to first order in $b(r)$ can be written as [23]

$$\bar{\rho}(\vec{r}) = \rho(r) + \int d\vec{r}_1\, b(|\vec{r}-\vec{r}_1|)\left[4\rho(\vec{r})\,\rho(\vec{r}_1) - |\rho(\vec{r},\vec{r}_1)|^2\right]$$

$$-\int d\vec{r}_1 d\vec{r}_2\; b(|\vec{r}_1-\vec{r}_2|)\,\rho(\vec{r},\vec{r}_1)\left[4\rho(\vec{r}_1,\vec{r})\,\rho(\vec{r}_2) - \rho(\vec{r}_1,\vec{r})\,\rho(\vec{r}_2,\vec{r})\right] \qquad \ldots \qquad (1.2.8)$$

Clearly, the charge density is just twice the density $\bar{\rho}(\vec{r})$.

The charge form factor is defined as

$$F(q) = \frac{2}{Z}\int d\vec{r}\; e^{i\vec{q}\cdot\vec{r}}\;\bar{\rho}(\vec{r}) \qquad \ldots \qquad (1.2.9)$$

and it is normalized such that

$$F(0) = 1 \, .$$

We assume that for open shell nuclei with partially filled shells the terms due to correlations are same as they would be in a closed shell nucleus with the only difference that the number of correlated pairs is now changed. We then obtain for a closed shell nucleus with $A = 2Z$,

$$F(q^2) = \frac{1}{A}\Big[\sum_{\alpha} \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle + \sum_{\alpha\beta} \langle \phi_{\alpha\beta} | e^{i\vec{q}\cdot\vec{r}_1}\, b(r_{12}) |\, \phi_{\alpha\beta} \rangle$$

$$- \sum_{\alpha\beta} \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle \langle \phi_{\alpha\beta} | b(r_{12}) | \phi_{\alpha\beta} \rangle], \quad \ldots \qquad (1.2.10)$$

where $\phi_{\alpha\beta}$ are properly antisymmetrized two particle uncorrelated wavefunctions and A is the number of nucleons.

Noting that

$$b = |f|^2 - 1 = (f-1)^2 + 2(f-1),$$

(1.2.10) can be written as

$$F(q^2) = \frac{1}{A} \sum_{\alpha} \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle + \frac{1}{A} \sum_{\alpha\beta} [\langle X_{\alpha\beta} | e^{i\vec{q}\cdot\vec{r}_1} | X_{\alpha\beta} \rangle$$

$$- \langle X_{\alpha\beta} | X_{\alpha\beta} \rangle \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle]$$

$$- \frac{2}{A} \sum_{\alpha\beta} [\langle \phi_{\alpha\beta} | e^{i\vec{q}\cdot\vec{r}_1} | X_{\alpha\beta} \rangle - \langle \phi_{\alpha\beta} | X_{\alpha\beta} \rangle \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle], \quad \ldots \qquad (1.2.11)$$

where

$$X_{\alpha\beta} = \phi_{\alpha\beta} - f \phi_{\alpha\beta}. \qquad \ldots \qquad (1.2.12)$$

Degli Atti and Kallio[39] call the first and the second terms within parentheses of (1.2.11) as contributions to the charge form factor arising due to 2p-2h and 1p-1h excitations respectively.

3. RIPKA, GILLESPIE AND GAUDIN'S CRITIQUE OF CHARGE FORM FACTOR CALCULATIONS WITH JASTROW WAVEFUNCTIONS.

Expressions (1.2.8) and (1.2.10) have been used by

several authors [4,7,31] to analyse elastic electron scattering experiments. It was observed that the high momentum transfer experiments were better accounted for with the correlated density than with the unperturbed density calculated with orbits of either harmonic oscillator or Wood-Saxon potentials. The need for the second and third terms in (1.2.8) and (1.2.10) was interpreted as giving information about short range correlations in the nucleus.

Rinker, Gillespie and Gaudin[23] have pointed out a disturbing feature of the analyses of elastic electron scattering data with Jastrow wavefunction. The low momentum transfer data, with $q < 2.5$ fm^{-1} can be accounted for quite satisfactorily with an uncorrelated density calculated with the orbits of Woods-Saxon[40] or harmonic oscillator potential But the Jastrow wavefunctions which were required to fit the high momentum transfer data had a Slater determinant with considerably different parameter values from the one required to fit the low q data. They "show" by an explicit calculation that it is possible to find a Slater determinant which can fit the low as well as high momentum transfer data. They claim that as far as charge density is concerned the main effect of Jastrow correlation factor acting on the original Slater determinant is to produce a new (uncorrelated) Slater determinant. As an example, they have calculated this new Slater determinant in case of ^{40}Ca. Their calculation is

briefly described below :

Firstly, the density matrix

$$\bar{\rho}_{ij} = \langle \Psi | a_j^\dagger a_i | \Psi \rangle \qquad \ldots \qquad (1.3.1)$$

is calculated with the Jastrow wavefunction (1.2.1) in the single particle basis of oscillator wavefunctions. The terms included in the calculation are upto second order in h(r) and first order in b(r). In terms of $\bar{\rho}_{ij}$ the charge density is

$$\bar{\rho}(\vec{r}) = \sum_{ij} \phi_i(\vec{r})\, \phi_j(\vec{r})\, \bar{\rho}_{ij} , \qquad \ldots \qquad (1.3.2)$$

where the sum is limited to proton single particle states $\phi_i(r)$.

The density matrix $\bar{\rho}_{ij}$ is then diagonalized

$$\sum_j \bar{\rho}_{ij}\, V_j^\alpha = n_\alpha\, V_i^\alpha \qquad \ldots \qquad (1.3.3)$$

in a space containing ^1s, ^2s, ^3s, ^4s; ^1p, ^2p, ^3p; ^1d, ^2d, ^3d; ^1f and ^1g single particle states. n_α are the eigenvalues and V_i^α the eigenvectors.

This yields a set of new orbits

$$\phi_\alpha^0(\vec{r}) = \sum_i V_i^\alpha\, \phi_i(\vec{r}) \qquad \ldots \qquad (1.3.4)$$

belonging to the new Fermi sea (F'). This is then utilized to construct a new Slater determinant ϕ^0 with the A orbits

belonging to eigenvalues which are close to unity. One can then separate the charge density into two terms,

$$\bar{\rho}(\vec{r}) = \rho^{0}(\vec{r}) + \rho_{c}(\vec{r}), \qquad \ldots \qquad (1.3.5)$$

where $\rho^{0}(\vec{r})$ is the charge density due to the new Slater determinant ϕ^{0} and $\rho_{c}(\vec{r})$ is the remainder which is defined as being due to the effect of correlations

$$\rho^{0}(\vec{r}) = \sum_{\alpha \in F'} | \phi_{\alpha}^{0}(\vec{r}) |^{2}$$

$$\rho_{c}(\vec{r}) = \sum_{\alpha \in F'}{}' n_{\alpha} | \phi_{\alpha}^{0}(\vec{r}) |^{2} - \sum_{\alpha \in F'}{}' (1-n_{\alpha}) | \phi_{\alpha}^{0}(\vec{r}) |^{2} \ldots \qquad (1.3.6)$$

The new Slater determinant has the following properties :

(i) It is independent of the basis functions used to calculate the density matrix $\bar{\rho}_{ij}$.

(ii) It will yield $\rho_{c} = 0$ if the state ψ is a Slater determinant.

Figure (1) shows charge densities of ^{40}Ca (uncorrected for the proton form factor and centre of mass motion) using a harmonic oscillater constant $a(|\frac{\hbar}{mw}|^{2}) = 1.82$ fm and a Jastrow function $1 - f^{2}(r) = \exp(-\beta^{2} r^{2})$ with $\beta = 1.4$ fm^{-1}. The dotted line shows the charge density ρ_{0} of the original Slater determinant ϕ made up of oscillator orbits. The solid line is the charge density, eq.(1.3.5), of the Jastrow wavefunction (1.2.1). The dashed line is the new charge

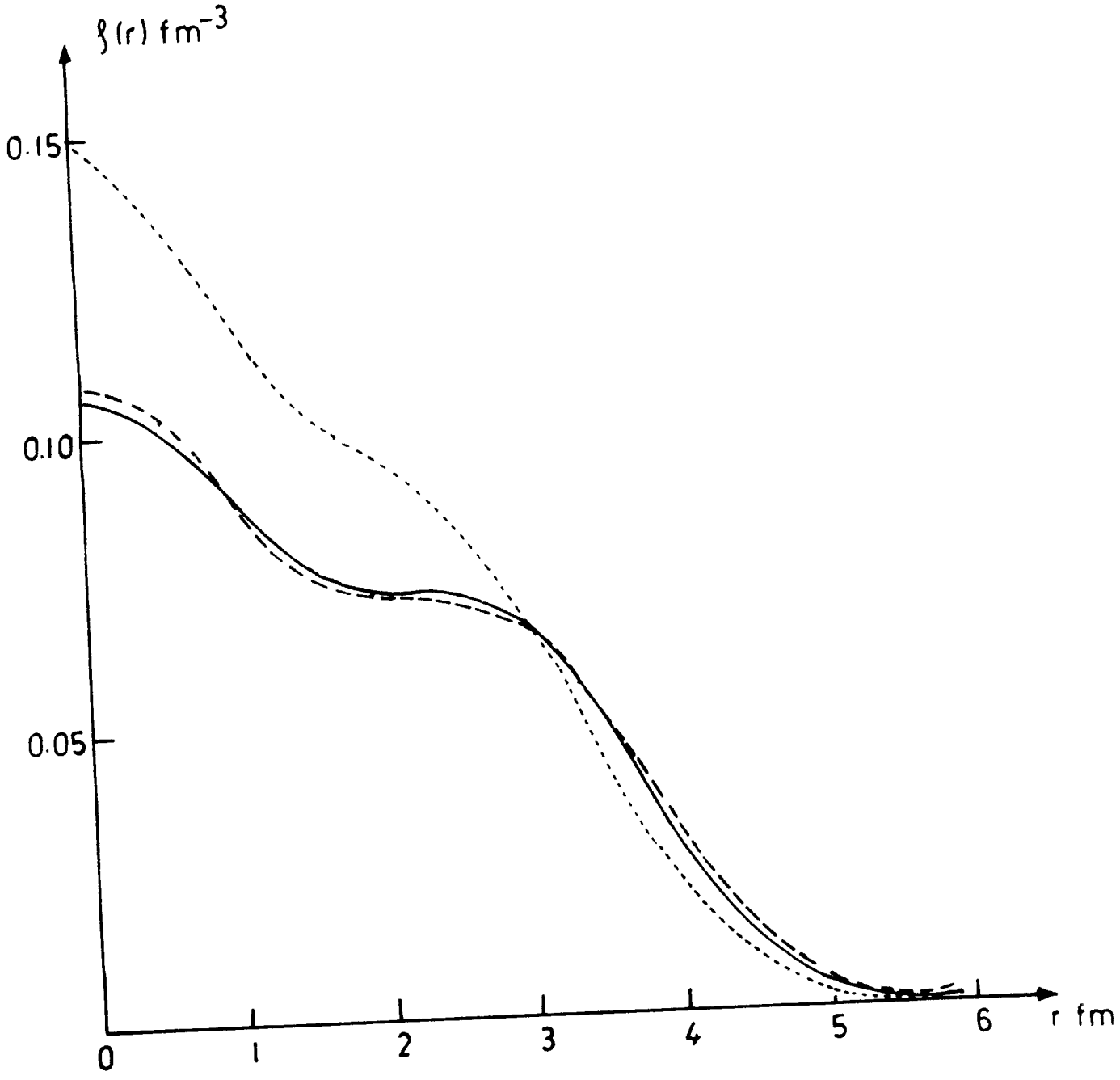

Fig. 1

density of the new Slater determinant ϕ^{o}. They have, therefore, concluded that since $\rho_c(\vec{r})$ is much smaller than the difference between the actual density and the uncorrelated density of the Slater determinant of the Jastrow wavefunction, it is no measure of the effect of correlations. However, this conclusion which seemed quite convincing on the face of it was controverted by Degli Atti. He demonstrated that even a small change in the density, i.e., a very small ρ_c, affects the charge form factor significantly in the high momentum transfer region, thus, raising doubt about the validity of the claim of Ripka et.al.[23]. In what follows we point out that the objection of Degli Atti can be fully met in the framework of Ripka et.al.[23].

The Jastrow correlation factor when operating on the unperturbed Slater determinant ϕ produces 1p-1h and 2p-2h excitations. Looking at the third term within the parentheses of expression (1.2.11) we see that only the 1p-1h component of the Jastrow wavefunction will contribute . The contribution due to 2p-2h excitations from this term will vanish because $e^{i\vec{q}\cdot\vec{r}_1}$ is a one body operator. The 2p-2h contributions which are considered a measure of the effect of short range correlations, will come through the second term within the parentheses. But the total contribution due to this term is quite small which contains some 1p-1h contribution also .

This has been found to be the case[23] in ^{40}Ca and in as light a nucleus as ^{4}He. Therefore, the bulk of the effect which is due to 1p-1h excitation can be taken care of by defining the new Slater determinant which yields a charge density very close to the correlated charge density.

If the function (f-1) were replaced by the operator $Q(f-1)$, where Q projects pairs of particles outside the Fermi sea as in the Brueckner theory, all the 1p-1h elements (to the order in which $\bar{\rho}_{ij}$ is calculated) would vanish; the diagonalization(1.3.4) would then yield orbits ϕ_α which are linear combinations of the occupied orbits in ϕ so that $\phi^o = \phi$. In such a situation the third term within the parentheses in (1.2.11) would vanish identically.

We notice that if we drop the second term within the parentheses of (1.2.11) the expression reduces to(1.2.10) with b(r) replaced by 2 h(r). Therefore, if elastic electron scattering data can be explained with (1.2.10), it can equally well be explained using (1.2.11) without the second term, and the contribution of 2p-2h excitation to the data in this situation will be identically zero. In such a situation $\rho_c(r)$ will also be identically zero. Therefore, keeping the cluster expansion only upto the first order in h(r) the objection of Degli Atti can be fully met. However, the present author has a more serious and valid criticism of the conclusions

of Ripka et.al.[23] from another point of view. This would be presented in the next section.

4. THE CENTRE OF MASS MOTION AND THE SHORT RANGE CORRELATIONS:

The criticism made in the preceding section regarding the extraction of short range correlation from elastic electron scattering data is doubtful because the important effect of the centre of mass motion has not been taken into account. Conclusions reached are probably not wrong for a relatively heavy nucleus like ^{40}Ca where the omission of the effect of centre of mass motion may not be inadequate in the experimentally explored momentum transfer range. However, for light nuclei, giving proper consideration to the centre of mass motion we will see that important 2p-2h contributions are present in the momentum transfer range of $0 \leq q \leq 4.5$ fm^{-1}. On the contrary, we have seen in the previous section that 2p-2h excitations will not contribute in the first order of h(r) where no consideration of the centre of mass motion was made. The importance of the centre of mass motion comes about with the realization that in elastic scattering all the A nucleons of the target must recoil together. This effect is included automatically in one, two and many body densities through the use of coordinates taken relative to the nuclear centre of mass. On the other hand, if Jastrow wavefunctions are

employed in which the Slater determinant ϕ does not depend on intrinsic coordinates, a centre of mass correlation which contains not only two body correlation but also three to A-body correlations must be considered. It then follows that the expectation value of an intrinsic operator becomes an expectation value of a many body operator. In particular, the expression for the charge form factor, in Born approximation, becomes

$$F(q^2) = \langle \Psi | e^{i\vec{q}.(\vec{r}_1 - \vec{R})} | \Psi \rangle . \qquad \ldots \qquad (1.4.1)$$

Expression (1.4.1) follows from the exact expression in Born approximation

$$F(q^2) = \langle \phi (\vec{\xi}_1, \ldots, \vec{\xi}_A) | e^{i\vec{q}.\vec{\xi}_1} \delta(\Sigma_i \vec{\xi}_i / A) | \phi (\vec{\xi}_1, \ldots \vec{\xi}_A) \rangle$$

$$\ldots \qquad (1.4.2)$$

with Ψ factorizable in intrinsic and centre of mass coordinates:

$$\Psi (\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A) = \phi (\vec{\xi}_1, \ldots \vec{\xi}_A) \Phi (\vec{R}), \qquad \ldots \qquad (1.4.3)$$

where

$$\vec{\xi}_i = \vec{r}_i - \vec{R}$$

and

$$\vec{R} = \Sigma_i \vec{r}_i / A .$$

As far as the removal of centre of mass motion is concerned,

expression (1.4.1) is exact if the Slater determinant ϕ of the Jastrow wavefunction consists of harmonic oscillator wavefunctions for which (1.4.3) holds.

From (1.4.1), it is seen that in particle-hole expansion of Ψ not only 2p-2h but also 3p-3h etc. shall contribute in the lowest order. To be more explicit we write the many body wavefunction in terms of the unitary model operator approach

$$| \bar{\Psi} > = e^{S} | \phi >, \qquad \ldots \qquad (1.4.4)$$

where the state ϕ represents the s.p. determinant and $S = \sum_{n=1}^{A} S_n$. The operator S_n excites n particle-n hole pairs. Substituting (1.4.4) into (1.4.1) we have

$$F(q^2) = < \phi | e^{\Sigma S_n^\dagger} O(\vec{r}_1, \vec{R}) e^{\Sigma S_n} | \phi >$$

$$= < \phi | O(\vec{r}_1, \vec{R}) | \phi > + < \phi | O(\vec{r}_1, \vec{R}) S_1 | \phi > + < \phi | S_1^\dagger O(\vec{r}_1, \vec{R}) | \phi >$$

$$+ < \phi | S_1^\dagger O(\vec{r}_1, \vec{R}) S_1 | \phi > + \ldots .$$

$$+ < \phi | O(\vec{r}_1, \vec{R}) S_2 | \phi > + < \phi | S_2^\dagger O(\vec{r}_1, \vec{R}) | \phi > + < \phi | S_2^\dagger (\vec{r}_1, \vec{R}) S_2 | \phi >$$

$$+ \ldots . \qquad \ldots \qquad (1.4.5)$$

The first term on the r.h.s in (1.4.5) is the unperturbed charge form factor. The matrix elements involving S_1 are the contributions due to 1p-1h excitations and those involving S_2 are the contributions due to 2p-2h excitations and so on.

It is seen that the terms involving either S_2 or $S_2^\dagger$ are not zero. Whereas, if we drop R from the exponential, the operator O becomes a single particle operator, the first two terms involving S_2 in (1.4.5) would be zero and the theorem of Fink et.al.[21] follows. To see how the matrix elements involving S_2 depend on mass number and momentum transfer we write

$$S_2 | \phi > = | \phi_{2p-2h} > . \qquad \ldots \qquad (1.4.6)$$

Neglecting the exchange we may obtain

$$< \phi | e^{i\vec{q}.(\vec{r}_1 - \vec{R})} S_2 | \phi > = <\phi | e^{i\vec{q}.(\vec{r}_1 - \vec{R})} | \phi_{2p-2h} >$$

$$= <\phi(1,2) | e^{i\vec{q}.\left(\frac{A-1}{A}\vec{r}_1 - \frac{1}{A}\vec{r}_2\right)} | \phi'(1,2)> <\phi(3,..,A)| e^{i\vec{q}.\frac{1}{A}}$$

$$(\vec{r}_3 + \ldots + \vec{r}_A) | \phi(3,..,A) > ,$$

where prime on ϕ (1,2) means that the states of particles 1 and 2 lie above the Fermi sea. Because of the orthogonality of states the matrix element

$$<\phi(1,2)| e^{i\vec{q}.\left(\frac{A-1}{A}\vec{r} - \frac{1}{A}\vec{r}_2\right)} | \phi'(1,2)> \ldots \qquad (1.4.7)$$

at finite momentum transfers shall go to zero as A tends to ∞. Therefore, if one wishes to see correlation effects in a massive nucleus one has to go to very high momentum transfers not accessible experimentally. On the other hand for light

nuclei the correlation effects might become important at relatively lower momentum transfers, thus, making their determination feasible.

We now write the Fourier transform of (1.4.5)

$$\bar{\rho}(\vec{r}) = \rho_0(\vec{r}) + \rho_{1p-1h}(\vec{r}) + \rho_{2p-2h}(\vec{r}) + \cdots, \ldots \quad (1.4.8)$$

where $\rho_0(\vec{r})$ is the Fourier transform of the first term of (1.4.5), ρ_{1p-1h} is the Fourier transform of the terms involing S_1 and ρ_{2p-2h} is that of S_2 etc. In (1.4.8) the contribution $\rho_0(\vec{r}) + \rho_{1p-1h}(\vec{r})$ is the part of the density that can be reproduced by self consistent wavefunction or can be obtained, to a large extent, by diagonalizing the density matrix as described in sec.3. It was found that the term ρ_{2p-2h} is quite small. This was because of the fact that when centre of mass effect is not taken into account the matrix element (1.4.7) is identically zero for all momentum transfers, which otherwise contributes to ρ_{2p-2h} at high momentum transfers even for a heavy nucleus. Though, it may be expected that the matrix element (1.4.7) may not contribute appreciably for a heavy nucleus in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}, but to argue that the elastic scattering is insensitive to correlations by calculating the density and diagonalizing the density matrix is not correct as these matrix elements for momentum transfers greater than 4.5 fm^{-1} may contribute appreciably through 2p-2h excitations. Therefore, the conclusions of Ripka et.al.[23] regarding the correlation effects

in ^{40}Ca are probably not wrong because these effects may not be present in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1} for such a heavy nucleus and not because of the fact that major part of the ^{40}Ca density can be accounted for by diagonalizing the density matrix (without the centre of mass effect).

When the separation of the nuclear wavefunction in terms of intrinsic and centre of mass coordinates is not possible, i.e. when (1.4.3) does not hold, one faces [34,35] the difficulty of nonuniqueness in removing the effect of centre of mass motion. For different prescriptions to remove the centre of mass motion one obtains different results. Therefore, all successful interpretations of the data in terms of single particle wavefunctions become meaningless. This problem is inherent in all translationally noninvariant wavefunctions when (1.4.3) does not hold. We show (cf appendix) that, as far as the single particle wavefunctions are concerned the harmonic oscillator wavefunctions are the only one, besides the wavefunctions of an ensemble of free particles, for which (1.4.3) holds. Though, it has been claimed that in principle it might be possible to carry out the calculations in a fairly accurate way with translationally noninvariant wavefunctions[41], but this is a very difficult task and to author's knowledge no calculation of this nature has so far been attempted.

When Jastrow wavefunctions are used with the Slater

determinant consisting of harmonic oscillator orbitals all prescriptions to remove the centre of mass motion lead to the same results, since (1.4.3) holds. Therefore, it seems practically feasible to extract information about short range correlations from elastic electron scattering by using Jastrow theory with the Slater determinant consisting of harmonic oscillator orbitals. Then, one has to carry out the particle hole expansion taking into account the centre of mass motion and compare the various contributions. If it turns out that 2p-2h excitations contribute significantly it would be a sufficient indication of the fact that elastic electron scattering is sensitive to short range correlations.

One generally expects that the contributions due to 1p-1h excitations can be taken care of by self consistent orbitals which are independent of the single particle basis function. Therefore, it should be found that the sum of the unperturbed and 1p-1h contributions is independent of the model single particle orbitals employed in the Slater determinant, which in our case shall correspond to independence with respect to the oscillator range parameter 'a'. We shall thus demonstrate the independence of the short range correlation effects with respect to the model single particle orbitals of the Slater determinant. This will have an important bearing on our conclusions.

In this section, what we have essentially tried to

emphasize is that the correlation contribution to elastic electron scattering starts from the first order in h(r) as against the general belief that it should start from the second order onwards. This, however, leads to a paradox. For Ψ given by (1.2.1) with ϕ consisting of harmonic oscillator orbitals only we have

$$\langle \Psi | e^{i\vec{q}.(\vec{r}_1-\vec{R})} | \Psi \rangle = \exp\left(\frac{q^2 a^2}{4A}\right) \langle \Psi | e^{i\vec{q}.\vec{r}_1} | \Psi \rangle \ldots \quad (1.4.9)$$

where $\exp(q^2 a^2/4A)$ is the well known Tassie-Barker correction due to centre of mass motion. Looking at the right hand side of (1.4.9) we realize that the 2p-2h excitations will start contributing from the second order, whereas, from the left hand side we see that 2p-2h contributions will start occuring from the first order itself. The above paradox may be resolved by realising that the tracnslationally invariant correlation factors excite only the two particle relative motion. This implies that 1p-1h excitations brought about by h(r) must be accompanied by 2p-2h excitations so that the centre of mass spuriousity present in the harmonic oscillator single particle wavefunction is not altered. This means that there exists a definite relationship between the 1p-1h and 2p-2h excitations which may be seen in the following way :

We write,

$$h(r_{12})|\phi\rangle = \sum_{1p-1h} \langle \phi_{1p-1h}|h|\phi\rangle |\phi_{1p-1h}\rangle + \sum_{2p-2h} \langle \phi_{2p-2h}|h|\phi\rangle |\phi_{2p-2h}\rangle \quad \ldots \quad (1.4.10)$$

where ϕ_{1p-1h} stands for the wavefunction in which at most one particle is outside the occupied states whereas in ϕ_{2p-2h} two particles are outside the occupied states. We now expand ϕ_{p-h} in terms of a finite sum of products of intrinsic and centre of mass wavefunctions [42]

$$|\phi_{p-h}\rangle = \sum_{nN} \langle nN|p-h\rangle |\phi_n(\vec{\xi}_1,\ldots,\vec{\xi}_A)\phi_N(\vec{R})\rangle, \ldots \quad (1.4.11)$$

where n and N correspond to quantum numbers of intrinsic and centre of mass motion respectively, while p - h corresponds to the quantum numbers of particle-hole states. The quantity $\langle n N|p-h\rangle$ represents the transformation brackets. Substituting (1.4.11) in (1.4.10) we get

$$h(r_{12})\phi\rangle = \sum_{1p-1h} \langle \phi_{1p-1h}|h|\phi\rangle \sum_{nN}\langle nN|1p-1h\rangle|\phi_n(\vec{\xi}_1,\ldots,\vec{\xi}_A)\phi_N(\vec{R})\rangle$$

$$+ \sum_{2p-2h} \langle\phi_{2p-2h}|h|\phi\rangle \sum_{nN}\langle nN|2p-2h\rangle\langle\phi_n(\vec{\xi}_1,\ldots\vec{\xi}_A)\phi_N(\vec{R})\rangle \ldots (1.4.12)$$

Contracting from the left with $\phi_n(\vec{\xi}_1,\ldots,\vec{\xi}_A)\,\phi_{N'}(\vec{R})$ for N' not belonging to the lowest state of the centre of mass motion we get, since on the l.h.s. of (1.4.12) no excited configuration of the centre of mass motion occurs

$$\sum_{1p-1h}\langle n'N'|1p-1h\rangle\langle\phi_{1p-1h}|h|\phi\rangle + \sum_{2p-2h}\langle n'N'|2p-2h\rangle\langle\phi_{2p-2h}|h|\phi\rangle = 0$$

$$\ldots \quad (1.4.13)$$

The set of equations (1.4.13) for all possible values of n'

and N' establishes the important fact that the matrix elements $\langle \phi_{2p-2h} |h| \phi \rangle$ are not independent of $\langle \phi_{1p-1h}|h| \phi \rangle$ and vice versa. Therefore, it is not possible to find a translationally invariant h(r) so that it may excite 2p - 2h and 1p - 1h states in an arbitrary manner.

If h(r) is to satisfy the requirements of Brueckner-Goldstone theory we must have

$$\langle \phi_{1p-1h}|h(r)|\phi \rangle = 0$$

for all ϕ_{1p-1h}. This gives

$$\sum_{2p-2h} \langle nN|2p-2h \rangle \langle \phi_{2p-2h}|h|\phi \rangle = 0.$$

We, therefore, have

$$\langle \phi_{2p-2h}|h| \phi \rangle = 0.$$

This means, it is not possible to construct an h(r) which excites only 2p-2h states and at the same time depends only upon the relative coordinate. Thus, h of Brueckner-Goldstone theory is translationally non-invariant i.e. $h_{B.G} = h(\vec{r},\vec{R})$. This makes the charge form factor calculations in Brueckner-Goldstone theory much more ambiguous. Because, now in addition to the removal of spuriosity from the single particle wave-function one has to remove it from $h(\vec{r},\vec{R})$ also. To authors knowledge no calculation of this nature has ever been attempted.

We now consider the relation (1.4.9). If we confine ourselves to terms first order in h(r), it is sufficient to consider the relation

$$e^{\frac{q^2 a^2}{4A}} \langle\phi|e^{i\vec{q}\cdot\vec{r}_1} h(r_{12})|\phi\rangle = \langle\phi|e^{i\vec{q}\cdot(\vec{r}_1-\vec{R})}h(r_{12})|\phi\rangle . \quad \ldots \quad (1.4.14)$$

The left hand side of (1.4.14) may be written as

$$e^{\frac{q^2 a^2}{4A}} \langle \phi|e^{i\vec{q}\cdot\vec{r}_1} h(r_{12})| \phi \rangle = e^{\frac{q^2 a^2}{4A}} \sum_{1p-1h} \langle\phi|e^{i\vec{q}\cdot\vec{r}_1}|\phi_{1p-1h}\rangle \langle\phi_{1p-1h} |h|\phi\rangle . \quad \ldots \quad (1.4.15)$$

Making use of (1.4.11), we obtain from (1.4.15) after some manipulation

$$e^{\frac{q^2 a^2}{4A}} \langle\phi|e^{i\vec{q}\cdot\vec{r}_1}h(r_{12})|\phi\rangle = \sum_{1p-1h} \sum_{n,n'} \langle \phi_o(\vec{\xi})|e^{i\vec{q}\cdot\vec{\xi}_1} \delta(\sum_i \vec{\xi}_i / A)| \phi_n(\vec{\xi})\rangle$$

$$\langle \phi_{n'}(\vec{\xi})|h(\xi_{12})\delta(\sum_i \vec{\xi}_i/A)| \phi_o(\vec{\xi})\rangle|\langle no|1p-1h\rangle\langle 1p-1h|n'0\rangle$$

$$+ e^{\frac{q^2 a^2}{4A}} \sum_{N\neq o} \langle nN|1p-1h\rangle\langle 1p-1h|n'o\rangle\langle\phi_o(\vec{R})|e^{i\vec{q}\cdot\vec{R}}|\phi_N(\vec{R})\rangle, \ldots (1.4.16)$$

where $\vec{\xi}$ stands for the set $[\vec{\xi}_1, \vec{\xi}_2, \ldots, \vec{\xi}_A]$.

Similarly, for the right hand side of (1.4.14) we obtain

$$\langle\phi|e^{i\vec{q}\cdot(\vec{r}-\vec{R})}h(r_{12})|\phi\rangle = \sum_{1p-1h}\sum_{nn'}\langle n0|1p-1h\rangle\langle 1p-1h|n'0\rangle$$

$$\langle\phi_o(\vec{\xi})|e^{i\vec{q}\cdot\vec{\xi}_1}\delta(\sum_1\vec{\xi}_1/A)|\phi_n(\vec{\xi})\rangle\langle\phi_{n'}(\vec{\xi})|h(\xi_{12})\delta(\sum_1\vec{\xi}_1/A)|$$

$$\phi_o(\vec{\xi})\rangle + \sum_{2p-2h}\langle\phi|e^{i\vec{q}\cdot(\vec{r}_1-\vec{R})}|\phi_{2p-2h}\rangle\langle\phi_{2p-2h}|h(r_{12})|\phi\rangle \quad \ldots(1.4.17$$

Comparing (1.4.16) and (1.4.17) we get

$$\sum_{2p-2h}\langle\phi|e^{i\vec{q}\cdot(\vec{r}_1-\vec{R})}|\phi_{2p-2h}\rangle\langle\phi_{2p-2h}|h(r_{12})|\phi\rangle =$$

$$e\,\frac{q^2a^2}{4A}\sum_{1p-1h}\sum_{n}\sum_{N\neq o}\langle nN|1p-1h\rangle\langle 1p-1h|n'0\rangle$$

$$\langle\phi_o(\vec{\xi})|e^{i\vec{q}\cdot\vec{\xi}_1}\delta(\sum_1\vec{\xi}_1/A)|\phi_n(\vec{\xi})\rangle\langle\phi_{n'}(\vec{\xi})|h(\xi_{12})\delta$$

$$(\sum_1\vec{\xi}_1/A)|\phi_o(\vec{\xi})\rangle\langle\phi_o(\vec{R})|e^{i\vec{q}\cdot\vec{R}}|\phi_N(\vec{R})\rangle. \quad \ldots(1.4.18)$$

From (1.4.16)it is seen that if single particle orbital constructed by taking into account the 1p-1h excitations, are thrown into a sum of products of intrinsic and centre of mass components,the first term gives contribution when the centre of mass motion is in ground state.The second term is due to the excitation of the centre of mass motion which may be taken as spurious in the context of the single particle picture of the nucleus. On the contrary,in Jastrow theory,the above term is not spurious as seen from(1.4.15) where its contribution is equal to the contribution

due to 2p-2h excitations. Thus, we may say that whatever spuriosity is introduced by the correlation factor through 1p-1h excitations is exactly taken care of by the 2p-2h excitations so that the total spuriousity introduced over the unperturbed single particle orbitals is zero. This gives rise to definite relationship between 1p-1h and 2p-2h excitations as is evident from (1.4.10), (1.4.16), (1.4.17) and (1.4.18). This makes the extraction of correlation from elastic electron scattering feasible.

5. COMMENTS ON EARLIER ANALYSES

In the last section we arrived at the conclusion that elastic electron scattering may be sensitive to short range correlations at least in light nuclei. Thus, we might get the impression that it should be possible to determine them unambiguously through elastic electron scattering. However, in earlier analyses[4,7,31] it has been found that several correlation factors give equally good fit to the data. This has been attributed to the fact that since elastic electron scattering involves a one body operator it measures the short range correlations indirectly. Nevertheless, it has been emphasised

repeatedly[43-44] that to explain the elastic electron scattering data, particularly of few body systems, it is necessary to assume strong repulsion between two nucleons at short distances. These potentials having a soft core or attractive in the core region have been found inadequate to explain the diffraction structure[43 44]. Also, in the phenomenological fitting of the data within the framework of Jastrow theory all the correlation factors which have been employed correspond to strong repulsion between two nucleons at short distances. However, the assertions that elastic electron scattering data indicate a strong repulsion between two nucleons at short distances and that it is insensitiv to the detailed shape of the correlation factor because the elastic scattering process involves one body operator may be refuted on the basis of following argument :

The data on elastic electron scattering is available upto a certain finite maximum momentum transfer q_{max}. Consequently, a point by point determination of the charge density which is the Fourier transform of charge form factor (available upto some finite q_{max}) is not possible. Such a problem should also arise when a phenomenological determination of the short range correlation is attempted. This can be seen

in the following way. The charge form factor in Jastrow theory upto the first order in $h(r)$ is given by

$$F(q^2) = \frac{1}{A}\Big[\sum_{\alpha}\langle\alpha|e^{i\vec{q}\cdot\vec{r}_1}|\alpha\rangle + 2\sum_{\alpha\beta}\langle\alpha\beta|h(r)e^{i\vec{q}\cdot\vec{r}_1}|\alpha\beta\rangle$$

$$- 2\sum_{\alpha\beta\gamma}\langle\alpha|e^{i\vec{q}\cdot\vec{r}_1}|\beta\rangle\langle\alpha\gamma|h|\beta\gamma\rangle\Big],$$

where we have completely neglected the effect of centre of mass motion. Assuming $h(r)$ to be sufficiently short range so that we may approximate

$$h(r)\phi(\vec{r}_1)\phi(\vec{r}_2) \simeq h(r)\,\phi(\vec{R})\,\phi(\vec{R}), \qquad \ldots \qquad (1.5.1)$$

where

$$\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$$

we obtain for ^4He

$$F(q^2) \quad \int d\vec{r}_1 e^{i\vec{q}\cdot\vec{r}_1}|\phi(\vec{r}_1)|^2 + 6\int d\vec{r}\; e^{i\frac{\vec{q}}{2}\cdot\vec{r}}h(r)\int d\vec{r}' e^{i\vec{q}\cdot\vec{r}'}|\phi(\vec{r}')|^2$$

$$-6\int d\vec{r}\,h(r)\int d\vec{r}_1 d\vec{r}_2\; e^{i\vec{q}\cdot\vec{r}_1}|\phi(\vec{r}_1)|^2|\phi(\vec{r}_2)|^2, \qquad \ldots \qquad (1.5.2)$$

The third term on the r.h.s in (1.5.2) goes rapidly to zero as q increases and accounts for the normalisations. From the second term we see that from an analysis of $F(q^2)$ we can determine, at the best, the Fourier transform of $h(r)$ upto

$q_{max}/2$. Therefore, the amplitudes of the Fourier components of h(r) with a wavelength smaller than $\lambda = 4\pi/q_{max}$ are not determined by experiment. Since generally, the data on charge form factor are available upto the momentum transfer of 5 fm^{-1}, nothing is known about Fourier components of h(r) with $\lambda < 2.6$ fm.

Thus, given a h(r) which can fit the data upto a certain q_{max} we can obtain a hierarchy of h(r) differing in their Fourier components with wavelengths less than $4\pi/q_{max}$. This means that different h(r) which are found to fit the data may be due to the limitation of availability of the data only in a finite range of momentum transfer and not because of the one body character of the elastic electron scattering process.

An h(r) corresponding to a hard core potential should behave in the following manner,

$$h(r) = -1, \quad 0 < r < r_{core}$$

$$h(r) \longrightarrow 0, \quad r \longrightarrow \infty, \qquad \ldots \qquad (1.5.3)$$

where r_{core} is the hard core radius. We may write h(r) as

$$h(r) = \int_0^{q_{max}} H(q) e^{i\vec{q}\cdot\vec{r}}\, d\vec{q} + \int_{q_{max}}^{\infty} H(q) e^{i\vec{q}\cdot\vec{r}}\, d\vec{q} \qquad \ldots \qquad (1.5.4)$$

$$= A(r) + B(r),$$

where H(q) is the Fourier transformof h(r) which is relevant upto q_{max} so far as elastic electron scattering is concerned. A(r) corresponds to the first integral in (1.5.4) and B(r) to the second. Since the asymptotic behaviour of H(q) determines the behaviour[45] in position space around r = o , and as H(q) is not determined beyond q_{max} , we can choose a B(r) such that h(r), for example, becomes positive for $r < r_{core}$ and at the same time fit the elastic electron scattering data with in q_{max} . A positive h(r) for small r corresponds to attractive potentials (no repulsion at short distances). This means that a two nucleon potential having no hard core or strong repulsion can also account for the elastic electron scattering data.

We have reached the above conclusions by assuming that the correlations are sufficiently short ranged so that (1.5.1) and (1.5.2) holds. Relaxing the assumption, however, does not change the conclusion. Indeterminacy in the one body density due to the availability of the data in a finite range of moment transfer can always be associated with the indeterminacy in the correlations.

From above considerations it is evident that the use

of model correlation factors may be misleading. Such a situation is also encountered in the determination of density. In fact, without additional physical assumptions, it is impossible to determine density from elastic electron scattering in a model independent way. We have seen that such a problem exists in the study of short range correlations and may not be related to the one body character of the scattering process. Such a problem will also arise in experiments which measure the two body density directly. We, therefore, consider it proper, unless we come across reasonable physical assumptions to give some shape to short range correlation, to completely abandon the notion of a phenomenological correlation factor and, instead, work in terms of its moments. This shall be elaborated more in the next chapter.

CHAPTER - II

In the last chapter we reached at the conclusion that it may be possible to extract information about short range correlations from elastic electron scattering. It is convenient to use Jastrow theory with the Slater determinant consisting of harmonic oscillator orbitals which does not introduce any ambiguity in the removal of centre of mass motion. In order to see the correlation effects, clearly, one has to carry out the particle-hole expansion of the Jastrow wavefunction. To do it as such is a very difficult task. One has to resort to cluster expansions. In section 1, closely following the technique of ref.[23], we carry out a linked cluster expansion to calculate the expectation value (1.3.1). Since the method employed is essentially the same as that of ref.[23], only essential parts of the calculation are described in detail. We also calculate the expectation value (1.3.1) in independent pair approximation. In section 2 we carry out the particle-hole expansion. This is done exactly for ^{4}He. For other nuclei an approximate procedure is given the validity of which is tested in the next chapter. In section 3 we propose a highly flexible correlation factor.

Such a correlation factor is found to be essential to obtain a single particle model independent information on short range correlations. In section 4 we give a prescription to calculate the matrix elements with the proposed correlation factor. In section 5 the detailed calculations of the matrix elements are carried out.

1. LINKED CLUSTER EXPANSION AND THE INDEPENDENT PAIR APPROXIMATION FOR THE CHARGE FORM FACTOR.

In this section, we shall obtain expressions for the form factor in linked cluster expansion as well as in independent pair approximation. In the first order approximation it turns out that the two approaches give identical results if the Jastrow correlation factor satisfies the average Pauli condition[46]

$$< \alpha\beta|(f-1)|\alpha\beta > = 0 \qquad \ldots \qquad (2.1.1)$$

for α and β belonging to occupied states.

For the sake of completeness we shall rewrite here the expressions for the charge form factor in Born approximation

$$F(q^2) = < \psi |e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}| \psi > , \ldots \qquad (2.1.2)$$

and the Jastrow wavefunction

$$= \frac{1}{\sqrt{N}} \prod_{2\le i<j\le A} f(r_{ij})\, \phi(X_1 \ldots X_A), \qquad \ldots \qquad (2.1.3)$$

where N is the normalisation constant.

As stated earlier, from the outset we shall consider $\phi(X_1 \ldots, X_A)$ to consist of harmonic oscillator orbitals only.

(a) THE LINKED CLUSTER EXPANSION :

Substituting the Jastrow wavefunction, (2.1.3), into the expression for the charge form factor (2.1.2), we obtain

$$F(q^2) = \frac{1}{N} \int e^{i\frac{A-1}{A}\vec{q}\cdot\vec{r}_1} \prod_{2\le i\le A} |f(|\vec{r}_1-\vec{r}_i|)|^2 \prod_{2\le i<j\le A} |f(r_{ij})|^2$$

$$|\phi(X_1, X_2, \ldots, X_A)|^2 e^{-\frac{1}{A}\sum_i \vec{q}\cdot\vec{r}_i}\, dX_1 dX_2 \ldots dX_A \ldots (2.14)$$

with the help of the functions

$$b(r) = f^2(r) - 1$$

and

$$h(r) = f(r) - 1 \quad ,$$

we may expand the product of the correlation factor in (2.1.4)

$$\prod_{2\le i\le A} f^*(|\vec{r}_1-\vec{r}_i|) f(|\vec{r}_1-\vec{r}_i|) \prod_{2\le i<j\le A} |1 + b(r_{ij})|$$

$$= 1 + \sum_{2\le i\le A} W_1(r_i) + \sum_{2\le i<j\le A} W_2(r_i, r_j) + \ldots\ldots \quad , \qquad (2.1.5)$$

where $W_p(r_1,\ldots,r_p)$ are symmetric p - body operators obtained by grouping all the terms which depend on the p variables. Two of the first W_p in (2.1.5) are

$$W_1(r_1) = h^*(|\vec{r}_1-\vec{r}_1|)+h(|\vec{r}_1-\vec{r}_1|)+|h(|\vec{r}_1-\vec{r}_1|)|^2$$

$$W_2(r_1,r_j)=b(r_{1j})+2h^*(|\vec{r}_1-\vec{r}_1|)h(|\vec{r}_1-\vec{r}_1|)+h^*(|\vec{r}_1-\vec{r}_1|)b(r_{1j})+\ldots$$

... (2.1.6)

Following relation can be shown to hold

$$\int|\phi(X_1,X_2,\ldots X_A)|^2 \exp(-\frac{i}{A}\sum_{i=2}^{A}\vec{q}.\vec{r}_i)\,dX_{p+1}\ldots dX_A$$

$$=(A-p)!\exp\left(-\frac{(A-(p+1))q^2a^2}{4A^2}\right)\exp\left(-\frac{i}{A}\sum_{i=2}^{p}\vec{q}.\vec{r}_i\right)\Delta_p(X_1,X_2,\ldots X_j$$

... (2.1.7)

where

$$\Delta_p(X_1,X_2\ldots X_p) = \det|\rho(X_i,X_j)|,\ i,j\in[1,p]$$

with $\rho(X_i,X_j) = \sum_{\alpha\in F}\phi^*_\alpha(X_i)\,\phi_\alpha(X_j)$.

The expansion (2.1.5) then yields the following expansion for the charge form factor

$$F(q^2) = \frac{A!}{N^A}\Big[\exp\Big(-\frac{(A-1)q^2a^2}{4A^2}\Big)\int\exp\Big(i\frac{A-1}{A}\vec{q}.\vec{x}_1\Big)\rho(x_1,x_1)dx_1$$

$$+ \frac{1}{1!}\exp\Big(-\frac{(A-2)q^2a^2}{4A^2}\Big)\int\exp\Big(-i\vec{q}.\Big(\frac{A-1}{A}\vec{x}_1 - \frac{1}{A}\vec{x}_2\Big)\Big)\rho_1(x_2)\Delta_2(x_1,x_2)$$

$$dx_1dx_2 + \frac{1}{2!}\exp\Big(-\frac{(A-3)q^2a^2}{4A^2}\Big)\int\exp\Big(-i\vec{q}.\Big(\frac{A-1}{A}\vec{x}_1 - \frac{1}{A}(\vec{x}_2+\vec{x}_3)\Big)\Big)$$

$$\rho_2(x_2,x_3)\,\Delta_3(x_1,x_2x_3)dx_1dx_2dx_3 + \ldots + \frac{1}{(A-1)!}\int\exp\Big(i\vec{q}\Big(\frac{A-1}{A}\vec{x}_1 - \frac{1}{A}\sum_{i=2}^{A}\vec{x}_i\Big)\Big)\rho_{A-1}\,\Delta_A dx_1\ldots dx_A\Big] \qquad \ldots \qquad (2.1.8)$$

Expression (2.1.8) is a finite sum of A integrals. If we define Δ_p for any value of p , we have

$$\Delta_p = 0 \quad \text{for} \quad p > A ,$$

as one cannot put more than A fermions into A different single particle orbits. Therefore, the sum (2.1.8) can be extended to infinity if Δ_p is defined for any value of p.

The calculation of charge form factor ,(2.1.8), follows almost the same line as the calculation of one body correlation function in ref.23). The various terms in the expansion of one body correlation function in terms of ρ_p consist of linked and unlinked parts. It has been shown [23] that all the unli

parts factor out and contribute the factor N/A!. Similarly, it may be shown that in (2.1.8) also the unlinked parts factor out and cancel the factor A!/N outside the brackets. We, therefore, obtain

$$F(q^2) = \frac{1}{A}\Big[\exp\Big(-\frac{(A-1)q^2a^2}{4A^2}\Big)\int\exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big)\rho(x_1,x_1)\,dx_1$$

$$+\sum_{p=1}^{\infty}\frac{1}{p!}\exp\Big(-\frac{A-(p+1)}{4A^2}q^2a^2\Big)\int\exp\Big(i\vec{q}.\Big(\frac{A-1}{A}\vec{r}_1-\sum_{i=2}^{p+1}\frac{\vec{r}_i}{A}\Big)\Big)$$

$$[V_p\,\Delta_{p+1}]\,dx_1..dx_{p+1}\Big]... \qquad (2.1.9)$$

where $[V_p\,\Delta_{p+1}]$ is the sum of all the linked parts contributing to $V_p\,\Delta_{p+1}$. If we take the whole of V_1 and $h^0(r_{ij})$ and $h(r_{ij})$ from V_2 and neglect all others the expansion (2.1.9) for a p-shell nucleus for A = 2Z becomes

$$F(q^2) = \frac{1}{A}\Big[\exp\Big(-\frac{(A-1)q^2a^2}{4A^2}\Big)\sum_{\alpha\in P}\langle\phi_\alpha|\exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big)|\phi_\alpha\rangle$$

$$+2\exp\Big(-\frac{(A-2)q^2a^2}{4A^2}\Big)\sum_{\alpha,\beta\in P}\langle\phi_{\alpha\beta}|\exp\Big(i\vec{q}.\Big(\frac{A-1}{A}\vec{r}_1-\frac{1}{A}\vec{r}_2\Big)\Big)h(r_{12})|\phi_{\alpha\beta}\rangle$$

$$-2\exp\Big(-\frac{(A-3)q^2a^2}{4A^2}\Big)\sum_{\alpha,\beta\in P}\langle\phi_\alpha|\exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big)|\phi_\alpha\rangle$$

$$\langle\phi_{\alpha\beta}|\exp\Big(\frac{i}{A}\vec{q}.(\vec{r}_1+\vec{r}_2)\Big)h(r_{12})|\phi_{\alpha\beta}\rangle\Big]... \qquad (2.1.10)$$

The first term in (2.1.10) is the shell model form factor

and the others are due to correlations.

(b) THE INDEPENDENT PAIR APPROXIMATION.

An alternative procedure, which has been employed in the past[31], is to expand the square of the product of correlation factors in terms of the number of b factors that it contains instead of grouping the terms according to the number of particles involved. The normalisation constant is also obtained in the same way. If the terms involving two and more b factors are neglected the approximation is known as independent pair approximation. Since the calculation of the charge form factor is straight forward in this approach we will give here only the final expression which reads as

$$F(q^2) \simeq \frac{1}{NA}\Big[\exp\Big(-\frac{(A-1)}{4A^2} q^2a^2\Big) \sum_{\alpha\in F} \langle \phi_\alpha | \exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big) | \phi_\alpha \rangle$$

$$+ \exp\Big(-\frac{(A-2)}{4A^2} q^2a^2\Big) \sum_{\alpha,\beta} \langle \phi_{\alpha\beta} | \exp\Big(i\vec{q}.\Big(\frac{A-1}{A}\vec{r}_1 - \frac{1}{A}\vec{r}_2\Big)\Big) b(r_{12}) | \phi_{\alpha\beta} \rangle$$

$$+ \exp\Big(-\frac{(A-3)}{4A^2} q^2a^2\Big) \sum_{\alpha} \langle \phi_\alpha | \exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big) | \phi_\alpha \rangle$$

$$\sum_{\substack{\beta,\gamma \\ \neq\alpha}} \langle \phi_{\beta\gamma} | b \exp\Big(\frac{i}{A}\vec{q}.(\vec{r}_2+\vec{r}_3)\Big) | \phi_{\beta\gamma} \rangle \Big] , \ldots \qquad (2.1.11)$$

where

$$N = 1 + \sum_{\alpha\beta} \langle \phi_{\alpha\beta} |b| \phi_{\alpha\beta} \rangle , \quad \ldots \qquad (2.1.12)$$

is the normalisation constant. The summation over α and β in the second term of (2.1.11) extends over the A occupied states whereas the summation over β and γ in the third term extends over the (A-1) occupied states.

Using the relation

$$b = |h(r)|^2 + 2\,h(r)$$

and neglecting terms second order in h(r) we obtain

$$F(q^2) = \frac{1}{N}\Big[\exp\Big(-\frac{(A-1)}{4A^2}q^2a^2\Big) \sum_{\alpha\in r} \langle \phi_\alpha | \exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big) | \phi_\alpha \rangle$$

$$+ 2\exp\Big(-\frac{(A-2)}{4A^2}q^2a^2\Big) \sum_{\alpha,\beta} \langle \phi_{\alpha\beta} | \exp\Big(i\vec{q}.\Big(\frac{A-1}{A}\vec{r}_1 - \frac{1}{A}\vec{r}_2\Big)\Big) h(r_{12}) | \phi_{\alpha\beta} \rangle$$

$$+ 2\exp\Big(-\frac{(A-3)}{4A^2}q^2a^2\Big) \sum_{\alpha} \langle \phi_\alpha | \exp\Big(i\frac{A-1}{A}\vec{q}.\vec{r}_1\Big) \phi_\alpha \rangle \sum_{\substack{\beta,\gamma \\ \neq \alpha}} \langle \phi_{\beta\gamma} | \exp\Big(\frac{i}{A}\vec{q}.$$

$$(\vec{r}_2 + \vec{r}_3)\Big) h | \phi_{\beta\gamma} \rangle \Big] \quad \ldots \qquad (2.1.13)$$

Expression (2.1.10) and (2.1.13) have been obtained in the two approaches upto the first order in h(r). They become identical if the auxiliary correlation factor satisfies the average Pauli condition (2.1.1). This is because the third

term in (2.1.10) and (2.1.13) become zero and the normalisation constant N becomes unity. It may be emphasised here that in earlier analyses[25,39], where the centre of mass effect was considered merely as a correction, the terms due to correlations in (2.1.10) and (2.1.13) have been claimed to be contributing via 1p-1h excitations only. On the contrary, it is obvious that 2p-2h excitations will also contribute to charge form factor in the first order of h(r).

2. SEPARATION OF PARTICLE-HOLE CONTRIBUTIONS.

In this section we separate out the contributions arising due to 1p-1h and 2p-2h excitations in the charge form factor (2.1.10). This is done exactly for ^{4}He. For other nuclei an approximate procedure is given which on the one hand facilitates the particle-hole separation in nuclei other than ^{4}He and on the other hand demonstrates that the contributions due to particle-hole excitations are independent of the prescriptions to remove the centre of mass motion.

(a) EXACT SEPARATION FOR ^{4}He.

For ^{4}He the charge form factor (2.1.10) may be

written as

$$F(q^2) = \exp(-\frac{3q^2a^2}{16}) + 6\exp(-\frac{q^2a^2}{32})$$

$$\langle \phi_{oo}(r_1)\,\phi_{oo}(r_2)|\exp(i\vec{q}.(\tfrac{3}{4}\vec{r}_1 - \tfrac{1}{4}\vec{r}_2))\,h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle$$

$$-6\exp(-\frac{3q^2a^2}{32})\langle\phi_{oo}(r_1)\phi_{oo}(r_2)|\exp(-i\vec{q}.(\frac{\vec{r}_1+\vec{r}_2}{4}))h(r_{12})|\phi_{oo}(r_1)$$

$$\phi_{oo}(r_2)\rangle \ldots \qquad (2.2.1)$$

The second and the third term in expression (2.2.1) can be interpreted as contributions to the charge form factor from 1p-1h and 2p-2h excitations in the nuclear wavefunction. The first term is due to the single particle harmonic oscillator wavefunction. In order to separate out the various particle -hole contributions we write

$$h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle = [(1-Q)+Q]h(r_{12})\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle,$$

$$\ldots \qquad (2.2.2)$$

where Q is the Pauli projection operator which projects out both the particles outside the occupied orbitals. The operator (1-Q), on the other hand, projects out at the most one particle outside the occupied orbitals. Therefore, the matrix elements corresponding to (1-Q) in the second and

the third term in expression (2.2.1) are contributions due to 1p-1h excitations.

We now expand $h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle$ in terms of the harmonic oscillator basic set

$$(1-Q)\, h(r_{12})\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle$$

$$=(1-Q)\sum_{n_1l_1}|\phi_{n_1l_1}(r_1)\phi_{n_2l_2}(r_2)\rangle\langle\phi_{n_1l_1}(r_1)\phi_{n_2l_2}(r_2)|h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle \quad \ldots \quad (2.2.3)$$

Applying (1-Q), we obtain

$$(1-Q)h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle=|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle\langle\phi_{oo}(r_1)\phi_{oo}(r_2)|h(r_{12})|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle$$

$$+\sum_{\substack{n_1l_1\\ n_1=l_1\neq o}}|\phi_{n_1l_1}(r_1)\phi_{oo}(r_2)\rangle\langle\phi_{n_1l_1}(r_1)\phi_{oo}(r_2)|h(r_{12})|\phi_{oo}(r_2)\rangle +$$

$$+\sum_{\substack{n_2l_2\\ n_2=l_2\neq o}}|\phi_{oo}(r_1)\phi_{n_2l_2}(r_2)\rangle\langle\phi_{oo}(r_1)\phi_{n_2l_2}(r_2)|h(r_{12})|\phi_{oo}(r_2)\rangle$$

Using the Moshinsky transformation[47] and invoking the energy and momentum conservations we get

$$(1-Q)h(r_{12})|\phi_{oo}(r_1)\,\phi_{oo}(r_2)\rangle=|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle$$

$$+\sum_{n\neq o}\frac{1}{2^n}\langle\phi_{oo}(r)|h(r)|\phi_{no}(r)\rangle[|\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle+|\phi_{oo}(r_1)\phi_{no}(r_2)\rangle] \quad \ldots \quad (2.2.4)$$

From (2.2.4)and the second and the third terms of

(2.2.1) we may now obtain the contribution to the charge form factor due to 1p-1h excitations.

For the second term we have

$$\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle\langle\phi_{oo}(r_1)\phi_{oo}(r_2)|\exp(i\vec{q}.(\tfrac{3}{4}\vec{r}_1-\tfrac{1}{4}\vec{r}_2))|$$

$$\phi_{oo}(r_1)\phi_{oo}(r_2)\rangle$$

$$+\sum_{n\neq o}\frac{1}{2^n}\langle\phi_{no}(r)|h(r)|\phi_{oo}(r)\rangle[\langle\phi_{oo}(r_1)|\exp(i\vec{q}.(\tfrac{3}{4}\vec{r}_1-\tfrac{1}{4}\vec{r}_2))|$$

$$\phi_{no}(r_1)\phi_{oo}(r_2)\rangle$$

$$+\langle\phi_{oo}(r_1)\phi_{oo}(r_2)|\exp(i\vec{q}.(\tfrac{3}{4}\vec{r}_1-\tfrac{1}{4}\vec{r}_2))|\phi_{oo}(r_1)\phi_{no}(r_2)\rangle\ ],$$

which may be easily reduced to

$$\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle\exp(-\frac{5q^2a^2}{32})+\sum_{n\neq o}\frac{1}{2^n}\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle$$

$$|\langle\phi_{oo}(r_1)|\exp(i\tfrac{3}{4}\vec{q}.\vec{r}_1)|\phi_{no}(r_1)\rangle\exp(-\frac{q^2a^2}{64})\langle\phi_{oo}(r_1)|$$

$$\exp(-i\tfrac{1}{4}\vec{q}.\vec{r}_1)|\phi_{no}(r_1)\rangle\exp(-\frac{9q^2a^2}{64})]\ldots(2.2.5a)$$

Similarly, for the third term we obtain

$$\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle\exp(-\frac{q^2a^2}{32})+\sum_{n\neq o}\langle\phi_{no}(r)|h(r)|\phi_{oo}(r)\rangle$$

$$\frac{1}{2^n}\exp(-\frac{q^2a^2}{64})$$

$$\langle\phi_{oo}(r)|\exp(\tfrac{1}{4}\vec{q}.\vec{r})|\phi_{oo}(r)\rangle\ .\qquad\ldots\ (2.2.5b)$$

Therefore, the sum of the second and the third terms corresponding to (1-Q), which we will call $F_{1p-1h}(q^2)$, after carrying out the integration involving the exponentials in (2.2.5) becomes

$$F_{1p-1h}(q^2) = 6 \sum_{n\neq o} \frac{1}{2^n} \langle \phi_{no}(r)|h(r)|\phi_{oo}(r)\rangle \left[\frac{2^n n!}{\Gamma(n+\frac{3}{2})\pi^{1/2}}\right]^{1/2}$$

$$\sum_{m=o}^{n} \frac{(-)^m}{m!} \binom{n+\frac{1}{2}}{n-m} \times$$

$$\times \Gamma(m+\frac{3}{2})\left[{}_1F_1(-m;\frac{3}{2};\frac{9q^2a^2}{64}) - {}_1F_1(-m;\frac{3}{2};\frac{q^2a^2}{64})\right]$$

$$\exp(-\tfrac{3}{16} q^2a^2), \qquad \ldots \qquad (2.2.6)$$

where ${}_1F_1$ are degenerate hypergeometric functions.

We will call $F_o(q^2)$ as the term due to op-oh excitation

$$F_o(q^2) = \exp\left(-\frac{3q^2a^2}{16}\right). \qquad \ldots \qquad (2.2.7)$$

The contribution due to 2p-2h excitations is obtained through the relation

$$F_{2p-2h}(q^2) = F(q^2) - F_o(q^2) - F_{1p-1h}(q^2), \ldots \quad (2.2.8)$$

where $F(q^2)$ is calculated directly from (2.2.1)

$$F(q^2) = \exp\left(-\frac{3q^2a^2}{16}\right) + 6\exp\left(-\frac{q^2a^2}{16}\right)\langle\phi_{oo}(r)|\exp(-1/2\ \vec{q}.\vec{r})|\phi_{oo}(r)\rangle$$

$$-6\exp\left(-\frac{3q^2a^2}{16}\right)\langle\phi_{oo}(r)|h(r)|\phi_{oo}(r)\rangle \ldots \quad (2.2.9)$$

(b) AN APPROXIMATE METHOD.

The approximate method to be described here can perhaps best be explained within the framework of unitary model operator approach. From (1.4.4), the charge form factor upto the first order of $(S_1 + S_2)$ can be written as

$$F(q^2) = \langle \phi|O(r_1,R)|\phi \rangle + \langle \phi|O(r_1,R)(S_1 + S_2)|\phi \rangle . \quad \ldots \quad (2.2.10)$$

The second term in (2.2.10) can be broken into two parts. The part corresponding to S_1 is 1p-1h excitation contribution while that corresponding to S_2 is the 2p-2h excitation contributions. We have separated the particle-hole contibutions according to this scheme for ^{4}He within the framework of Jastrow theory. We now give here a prescription to separate the particle-hole excitation contributions in an approximate way. We have seen that 2p-2h excitations contribute because of the presence of the coordinate R in the operator O. We write

$$\langle\phi|O(r_1,R)(S_1 + S_2)|\phi\rangle = \langle\phi|O(r_1)(S_1 + S_2)|\phi\rangle + \langle\phi|[O(r_1,R)-O(r_1)](S_1 + S_2)|\phi\rangle \quad \ldots \quad (2.2.11)$$

We shall interpret the first term on the r.h.s. of (2.2.11)

as contribution due to 1p-1h excitations while the second term as contribution due to 2p-2h excitations. Its advantages are that to [illegible] excitation contribution one has to calculate only the correlated part of the form factor without the centre of mass coordinate, since

$$\langle \phi | O(r_1)(S_1 + S_2) | \phi \rangle = \langle \phi | O(r_1) S_1 | \phi \rangle \quad \ldots \quad (2.2.12)$$

To find the 2p-2h excitation contribution one has to calculate the correlated part of the form factor with and without the centre of mass effect included and subtract the two. These calculations are much simpler to carry out than the exact separation, particularly for nuclei other than ^4He.

This procedure, however, introduces some errors. The second term of (2.2.11) contains some 1p-1h excitations also. Nevertheless, in the low momentum transfer region, its contributions can be expected small compared to (2.2.12) which is the dominant 1p-1h excitation term in this range of the momentum transfer. This can be seen as follows. Because of (2.2.12) the second term of (2.2.11) may be written as

$$\langle \phi | [O(r_1, R) - O(r_1)](S_1 + S_2) | \phi \rangle = \langle \phi | O(r_1, R) S_2 | \phi \rangle + \langle \phi | [O(r_1, R) - O(r_1)] S_1 | \phi \rangle \quad \ldots \quad (2.2.13)$$

Obviously,

$$O(r_1,R)-O(r_1)=O(r_1)[\cos(\vec{q}.\vec{R})+ i \sin(\vec{q}.\vec{R})- 1].$$

Since the integration over $\sin(\vec{q}.\vec{R})$ will vanish, we will obtain after expanding the $\cos(\vec{q}.\vec{R})$

$$O(r_1,R)-O(r_1)= - O(r_1)[\frac{(\vec{q}.\vec{R})^2}{2!} - \frac{(\vec{q}.\vec{R})^4}{4!} + ...].$$

Therefore, at low momentum transfers the second matrix elements on r.h.s of (2.2.13) will go to zero faster by a factor of momentum transfer squared than the matrix element in (2.2.12). However, the two matrix elements on the r.h.s. of (2.2.13) may be comparable. Thus, the error introduced in interpreting (2.2.13) as contribution due to 2p-2h excitations only may not be small at low momentum transfers. But, this large error is of not much significance in this range of momentum transfer as the whole of the second term in (2.2.11) will be much smaller than the first term. In the high momentum transfer region it is not unreasonable to expect that 2p-2h excitation terms would dominate. However, the error introduced in the interpretation, because of the second term in (2.2.13) which arises from 1p-1h excitations may not be small. But it would hardly matter since this term would anyway be quite

small compared to 2p-2h contributions. Thus we may expect that in the two ranges of momentum transfers those terms would be correctly predicted in our approximate procedure where they are dominant. In the intermediate region of momentum transfers where the contributions $F_0 + F_{1p-1h}$ (where F_0 is given by the first term on the r.h.s. of (2.2.11)) and F_{2p-2h} may compete our approximation could break down. All these expectations are borne out by actual calculations as we shall see in the next chapter where we have compared the exact and approximate separations for ^4He.

For (2.1.2) and (2.1.3) the approximate particle-hole separation may be written as

$$F(q^2) = \langle \Psi | \exp(i\vec{q}.\vec{r}_1) | \bar{\Psi} \rangle + \langle \phi | \exp(i\vec{q}.(\vec{r}_1 - \vec{R})) - \exp(i\vec{q}.\vec{r}_1) | \phi \rangle$$

$$+ \langle \bar{\Psi} | \exp(i\vec{q}.(\vec{r}_1 - \vec{R})) - \exp(i\vec{q}.\vec{r}_1) | \bar{\Psi} \rangle -$$

$$- \langle \phi | \exp(i\vec{q}.(\vec{r}_1 - \vec{R})) - \exp(i\vec{q}.\vec{r}_1) | \phi \rangle . \quad \ldots \quad (2.2.14)$$

The sum of the first two terms is $F_0 + F_{1p-1h}$ while the sum of the other two terms is due to particle-hole excitations other than 1p-1h. If $\bar{\Psi}$ contains excitations only upto 2p-2h the sum of the last two terms is F_{2p-2h}. The correlation

factors, being translationally invariant, preserve the Tassie-Barker correction. We therefore obtain from (2.2.14)

$$F_o + F_{1p-1h} \simeq \langle \Psi | \exp(i\vec{q}.\vec{r}_1) | \Psi \rangle + \langle \phi | (\exp(\frac{q^2a^2}{4A} - 1)) \exp(i\vec{q}.\vec{r}_1) | \phi \rangle, \quad \ldots \quad (2.2.15a)$$

$$F_{2p-2h} + \ldots + F_{Ap-Ah} = \langle \Psi | (\exp(\frac{q^2a^2}{4A} - 1) \exp(i\vec{q}.\vec{r}_1) | \Psi \rangle - \langle \phi | (\exp(\frac{q^2a^2}{4A} - 1)) \exp(i\vec{q}.\vec{r}_1) | \phi \rangle \ldots \quad (2.2.15b)$$

The approximate separation (2.2.15) have the following qualities :-

(1) To carry out the particle-hole separation with (2.2.15) is quite easy. Therefore, for nuclei other than ^{4}He where exact separation is not so easy (2.2.15) becomes useful.

(2) It may be noted that (2.2.15) is independent of the way the centre of mass motion is removed. This demonstrates that the particle-hole contributions, in particular $F_o + F_{1p-1h}$, are independent of hierarchy of prescriptions to remove the centre of mass motion.

(3) It demonstrates the important fact that the

approximate prescription (2.2.15) is independent of the order to which the cluster expansion of Ψ is made. This may be seen in the following way. Since, for a given set of harmonic oscillator single particle orbitals

$$F(q^2) = F_o + \exp\left(\frac{q^2a^2}{4A}\right) \bar{F}_{corr},$$

where

$$\bar{F}_{corr.} = \langle \Psi | \exp(i\vec{q}.\vec{r}_1) | \Psi \rangle - \langle \phi | \exp(i\vec{q}.\vec{r}_1) | \phi \rangle ,$$

is the correlated part of the form factor without the removal of centre of mass motion. Thus, in a precise fitting of the data $F_{corr.}$ and $\bar{F}_{corr.}$ remain same no matter to what order we carry out the cluster expansion. Therefore, it follows that (2.2.15a) and (2.2.15b) are also independent of the order to which the cluster expansion is made. However, the correlation parameters (or correlation factor) may become different for different orders of the cluster expansion.

3. PROPOSAL OF A CORRELATION FACTOR.

In the last section of chapter I we reached at the conclusion that the use of Gaussian or spherical Bessel

function type correlation factors may be quite misleading. Although, one may learn a few things with these model correlation factors, but the limitations inherent in them should not be overlooked. Results obtained are model dependent and the question arises whether they are required by the data or whether they are just a consequence of being too restricted in form. This was discussed in detail in section 5 of chapter I. Further, with the model correlation factors one is forced to vary the parameters of the single particle orbitals in order to achieve an adequate fit [23]. Consequently, one may obtain single particle orbitals which may be very different from self consistent or maximum overlap orbitals as defined by Kobe [48]. This may give rise to convergence problem in the cluster expansion and the conclusions obtained this way might be erroneous. Also, starting from a set of arbitrary single particle orbitals one should be able to find a correlation factor which fits the experimental charge form factor data. This follows from the fact that Jastrow correlation factor scatters particles into all states and not only into states in which both the particles are outside the occupied orbitals.

However, one must find that the separation of $F(q^2)$ into $F_o + F_{1p-1h}$ and F_{2p-2h} is independent of the single particle orbitals. In our case this corresponds to independence with respect to the oscillator range parameter 'a' . Therefore, in order to show the independence of $F_o + F_{1p-1h}$ with respect to the single particle orbitals and determine the short range correlation in a single particle independent way one needs a correlation factor of high flexibility. Such an objective cannot be accomplished with Gaussian or spherical Bessel function type correlation factors.

In order to overcome the above problems we shall work in terms of the moments of h(r). We shall show that correlation factors of arbitrary range and shape can be developed in terms of a series of velocity dependent correlation factors, in increasing powers of the momentum, but all of zero range. The coefficients of the terms in this series are the successive moments of the correlation factor , so that for short range correlation factors only the first few terms of the series will contribute significantly to the matrix elements involved. These matrix elements will also be easy to evaluate, as the velocity dependent correlation factors , being of zero range, contains

a $\delta(\vec{r}_1-\vec{r}_2)$ function. We shall first illustrate it by a one dimensional example.

Consider a one dimensional correlation factor h(x) of non zero range. In the momentum space representation it will be given by the operator

$$H(k'-k'') = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp(-ik'x)h(r)\exp(ik''x)dx \quad \ldots \quad (2.3.1)$$

Expanding the exponentials we obtain

$$H(k'-k'')=(2\pi)^{-1} \sum_{n'=0}^{\infty} \sum_{n''=0}^{\infty} i^{n''-n'} [n'!n''!]^{-1} I_{n'+n''} (k')^{n'} (k'')^{n''}$$

$$\ldots \qquad \ldots$$

$$\ldots \qquad (2.3.2)$$

where

$$I_{n'+n''}=\int_{-\infty}^{\infty} h(x)x^{n'+n''} dx$$

are the moments of h(x).

We assume, of course, that h(x) is of short range so that I 's are bounded.

We now construct in configuration space the zero range velocity dependent correlation factor

$$\sum_{n'=0}^{\infty} \sum_{n''=0}^{\infty} i^{n''-n'} [n'!n''!]^{-1} I_{n'+n''} k^{n'} \delta(x) k^{n''}, \ldots (2.3.3)$$

where k is the operator

$$k = -i \frac{\partial}{\partial x}.$$

The operator (2.3.3) has in momentum space the representations (2.3.2). Therefore h(x) and (2.3.3) have the same representation in momentum space, implying that they are also identical in any other representation. Therefore, a short range correlation factor can be developed in a series of velocity dependent correlation factors in increasing powers of the momentum, but of zero range.

Now let us take a short range correlation factor h(r) in three dimensions and consider its representation in momentum space

$$H(\vec{k}'-\vec{k}'') = (2\pi)^{-3} \int \exp(-i(\vec{k}'.\vec{r}))h(r)\exp(i(\vec{k}''.\vec{r}))d\vec{r}.$$

Integrating over the angular variables and expanding the sine function, we obtain

$$H(\vec{k}'-\vec{k}'') = (2\pi)^{-3} \int_0^\infty (4\pi h(r))(|\vec{k}'-\vec{k}''|r)^{-1}\sin(|\vec{k}'-\vec{k}''|r)dr$$

$$=(2\pi)^{-3}4\pi[I_1-(I_2/3!)(\vec{k}'-\vec{k}'')^2+(I_3/5!)(\vec{k}'-\vec{k}'')^4 - \ldots], \quad \ldots \quad (2.3.4)$$

where I's are the moments of the correlation factor given by

$$I_n = \int_0^\infty h(r)\, r^{2n}\, dr\,, \quad \ldots \quad (2.3.5)$$

for $n = 1,2,\ldots$ etc..

h(r) can now be represented by

$$h(r) = I_1\ 4\pi\ \delta(\vec{r}) - (I_2/3!)4\pi[k^2\delta(\vec{r}) + \delta(\vec{r})k^2 - 2\vec{k}.\delta(\vec{r})\vec{k}\ ]$$
$$+ (I_3/5!)4\pi[k^4\delta(\vec{r}) + \delta(\vec{r})k^4 + 2k^2\delta(\vec{r})\ k^2$$
$$- 4k^2\vec{k}.\delta(\vec{r})\vec{k} - 4\vec{k}.\delta(\vec{r})\ \vec{k}\ k^2 + 4\vec{k}.(\vec{k}.\delta(\vec{r})\vec{k})\ \vec{k}] \cdots \quad (2.3.6)$$

The expansion (2.3.6) shall be taken as the definition of correlation factor with I_n as its parameters.

Since the momentum transform of (2.3.6) is a Taylor series expansion of the correlation factor in momentum space around $q = 0$, (2.3.4), and if it is found that only the first few moments are necessary to fit the data, it follows that radial behaviour of h(r) is not determined for lower values of internucleon distances [45]. However, in order to have some qualitative idea about the radial behaviour of h(r) we write

$$h(r) = \sum_n a_n e^{-nr} \qquad \ldots \qquad (2.3.7)$$

$$h(0) = \sum_n a_n = 0 \qquad \ldots \qquad (2.3.8)$$

where the summation in (2.3.7) and (2.3.8) is extended upto

the number of moments necessary to fit the data plus one. C is an arbitrary constant which determines h(r) at r = 0 . For example, C = - 1 corresponds to the situation in which the two nucleon force is infinitely repulsive at the origin. The coefficients a_n can be determined from the values of the moments, employing eqn.(3.3.7) and the condition (3.3.8).

4. EVALUATION OF RADIAL MATRIX ELEMENTS WITH OUR PROPOSED CORRELATION FACTOR.

We shall encounter radial matrix elements of the form

$$< \phi_{nl}(r)|\ h(r)|\ \phi_{n'l}(r) > ,$$

$$< \phi_{nl}(r)|J_0(qr)\ h(r)|\ \phi_{n'l}(r) > ,$$

and $$< \phi_{nl}(r)|J_k(qr)\ h(r)|\ \phi_{n'l+2}(r) >, \ldots \quad (3.4.1)$$

for k = 0 and 2. The $\phi_{nl}(r)$ is the radial part of the harmonic oscillator wavefunction. Straightforward way to evaluate (3.4.1) is to substitute h(r) from (3.3.6), operate on $\phi_{nl}(r)$ and $J_k(qr)$ the $\hat{K}$, $\hat{K}^2$ etc. and then carry out the integration. There is a much simpler way also. We notice that in (3.4.1) the integrand, which we shall denote by G(q,r), apart from h(r) is an even function of r. Making a Taylor

series expansion of $G(q,r)$

$$G(q,r) = G(q,0) + r^2 \frac{\partial^2 G(q,r)}{\partial r^2}\Big|_{r=0} + \cdots\cdots$$

we obtain from (3.4.1)

$$\int G(q,r)\, h(r) r^2 dr = G(q,0) \int h(r)\, r^2 dr + \frac{\partial^2 G(q,r)}{\partial r^2}\Big|_{r=0} \int h(r)\, r^4 dr + \cdots .$$

Therefore,

$$\langle \phi_{n1}(r) | h(r) | \phi_{n'1}(r) \rangle = a_1 I_1 + a_2 I_2 + a_3 I_3 + \cdots , \quad (3.4.2)$$

where

$$a_1 = G(q,0)$$

$$a_2 = \frac{\partial^2 G(q,r)}{\partial r^2}\Big|_{r=0}$$

etc.,

with similar expression for other matrix elements in (3.4.1).

On the other hand if we evaluate directly (2.4.1) with (2.3.6) as mentioned in the beginning we would get

$$\langle \phi_{n1}(r) | h(r) | \phi_{n'1}(r) \rangle = f(I_1, I_2 \cdots)$$

$$= b_1 I_1 + b_2 I_2 + b_3 I_3 + \cdots \quad (2.4.3)$$

From (3.4.2) and (3.4.3) we obtain

$$a_1 I_1 + a_2 I_2 + a_3 I_3 + \ldots = b_1 I_1 + b_2 I_2 + b_3 I_3 + \ldots \quad (2.4.3).$$

Relation (2.4.3) must hold for every h(r). This means that the coefficients of each moments on the two sides of (2.4.3) must be equal i.e. $a_1 = b_1$, $a_2 = b_2$, etc.. We, thus see that we can very easily evaluate the matrix elements (2.4.1) by taking the Taylor series expansion of the integrand apart from h(r). Such a procedure will be adopted in the present work.

5. DETAILS OF CALCULATION.

The problem of calculating the charge form factor , particularly for nuclei other than the ^{4}He, in Born approximation is essentially the evaluation of the diagonal matrix elements of $h(r_{ij})$ and $G = e^{i\vec{q}.\vec{r}}\ h(r_{ij})$ in two particle states. Once this is done, the summation in (2.1.10) and (2.1.13) can be carried out by expressing the two particle states $| \phi_{\alpha\beta} >$ in terms of states with definite orbitals, spin and isospin quantum numbers λ , S and T respectively, with the help of Clebsch-Gordan coefficients. For closed shell nuclei the summation can be carried out exactly, whereas for open shell nuclei we shall assume the same results as that for the closed shell except with a difference that the number of

correlated pairs is now changed. However, this result would be exact for nuclei where the numbers of neutrons and protons are equal.

When considering the correlation pairs in the summation (2.1.10) and (2.1.13) we shall distinguish the two cases, namely, (a) those belonging to the same shell, and (b) the cross pairs belonging to different shells. In our case, this will correspond to the situation in which one of the nucleons is moving in s-shell while the other in p-shell .

The shell model states we are interested in are the well known harmonic oscillator radial functions

$$R_{0s} = \frac{2}{\pi^{1/4} a^{3/2}} e^{-r^2/2a^2},$$

$$R_{0p} = \frac{\sqrt{8}}{\sqrt{3}} \frac{1}{\pi^{1/4} a_p^{5/2}} r e^{-r^2/2a_p^2}.$$

We shall now evaluate the diagonal matrix elements of h and G and carry out the summation as indicated.

(a) For the shell model states characterized by the harmonic oscillator wavefunction moving in the same oscillator well the two particle states can be separated into relative and centre of

mass motion by means of Talmi-Moshinsky transformation and the matrix element of h(r) becomes

$$\langle n_1 l_1, n_2 l_2; \lambda\mu \,|h(r)|\, n_1 l_1, n_2 l_2; \lambda\mu - n_2 l_2, n_1 l_1; \lambda\mu \rangle$$

$$= \sum_{\substack{NL \\ nl}} |\langle n_1 l_1, n_2 l_2; \lambda\mu \,|NL, nl; \lambda\mu\rangle|^2 \langle nl||h||nl\rangle, \ldots \quad (2.5.1)$$

where $\langle n_1 l_1, n_2 l_2; \lambda\mu \,|NL, nl; \lambda\mu\rangle$ are Talmi-Moshinsky brackets and are zero if $l + S + T$ is even.

Similarly the matrix element of G, using Talmi-Moshinsky transformation, Racah algebra and the Wigner-Eckart theorem, can be put in the form

$$\langle n_1 l_1, n_2 l_2; \lambda\mu \;(S\;T)|G\,| \ldots\ldots \rangle$$

$$= \sum_{\substack{NLnl \\ N'L'n'l'}} \langle n_1 l_1, n_2 l_2; \lambda\mu \,|NL, nl\,; \lambda\mu\rangle\langle n_1 l_1, n_2 l_2; \lambda\mu \,|NL, nl; \lambda\mu\rangle$$

$$\sum_k W(L'k\,l\,;Ll')(2k+1)\left[(2L'+1)(2l'+1)\right]^{1/2} C(L'kL,ooo)C(l'kl,ooo)$$

$$\langle nl||h(r)\,j_k\left(\tfrac{1}{2}qr\right)||n'l'\rangle\langle NL||\,J_k(qR)||N'L'\rangle \ldots \quad (2.5.2)$$

where W(abcd;ef) and C (abc,ooo) are Racah and Clebsch-Gordan coefficients respectively.

The evaluation of the matrix elements (2.5.1) and (2.5.2) can be easily carried out with the energy and momentum

conservation in Talmi-Moshinsky brackets

$$\vec{l}_i + \vec{l}_j = \vec{\lambda} \equiv \vec{L} + \vec{l}$$

and $2n_i + l_i + 2n_j + l_j = 2N + L + 2n + l = 2N' + L' + 2n' + l'$.

(b) If we are dealing with particles moving in different wells for $n_i l_i \neq n_j l_j$, the usual Talmi-Moshinsky brackets cannot be used. However, we can express the matrix elements in terms of Talmi-integrals (for $n_i l_i = 0s$, $n_j l_j = 0p$) F^0 and G^1 by defining the centre of mass and relative coordinates as

$$\vec{R} = \frac{a_2^2 \vec{r}_1 + a_1^2 \vec{r}_2}{a_1^2 + a_2^2} \quad , \quad \vec{r} = \vec{r}_1 - \vec{r}_2 .$$

We obtain

$$\langle 0s, 0p; 1 \mid \mu(sT) \mid h \mid \ldots \rangle = \begin{cases} F^0 + \frac{1}{3} G^1 & \text{if } S + T \text{ is odd} \\ F^0 - \frac{1}{3} G^1 & \text{if } S + T \text{ is even} , \end{cases}$$

where

$$F^0 = \frac{1}{a_s^2 + a_p^2} \left[a_s^2 M_s(\vec{a}) + a_p^2 M_p(\vec{a}) \right]$$

$$G^1 = 24 \frac{a_s^5 a_p^3}{(a_s^2 + a_p^2)^4} \left[M_s(\vec{a}) - M_p(\vec{a}) \right] \ldots \qquad (2.5.3)$$

and

$$\langle 0s, 0p; 1 \mid \mu(sT) \mid G \mid \ldots \rangle = \begin{cases} F^0 + \frac{1}{3} G^1 \\ F^0 - \frac{1}{3} G^1 , \end{cases}$$

where

$$F^0 = \frac{1}{a_s^2 + a_p^2}\left[a_s^2 M_p(\bar{\bar{a}}) M_s(\bar{a}) + a_p^2 M_s(\bar{\bar{a}}) M_p(\bar{a}) \right]$$

$$G^1 = 24 \frac{a_s^5 a_p^3}{(a_s^2 + a_p^2)^4} \left[M_p(\bar{\bar{a}}) M_p(\bar{a}) - M_s(\bar{\bar{a}}) M_p(\bar{a}) \right]. \qquad (2.5.4)$$

The transformed oscillator lengths are as follows

$$\bar{a} = \sqrt{\frac{1}{2}} (a_s^2 + a_p^2)^{1/2} \quad , \quad \bar{\bar{a}} = \frac{\sqrt{2}\, a_s a_p}{(a_s^2 + a_p^2)^{1/2}}$$

In expression (2.5.3) the various $M_l(a)$ are the usual Talmi integral having the form

$$M_l(a) = N_l(a\sqrt{2})^2 \int \exp(-r^2/2a^2) h(r) r^{2l+2} dr. \ldots \qquad (2.5.5)$$

where, in the normalisation constant N_l, a is replaced by a $\sqrt{2}$.

Similarly, in expression (2.5.4) the various $M_{l(L)}(a)$ are

$$M_L(a) = N_L(a/\sqrt{2})^2 \int \exp(-2R^2/a^2) J_0(qR) R^{2L+2} dR \ ,$$

$$M_l(a) = N_l(a\sqrt{2})^2 \int \exp(-r^2/a^2) h(r) J_0(\tfrac{1}{2}qr) r^{2l+2} dr \ldots (2.5.6)$$

Having obtained the matrix element of h.G we can now carry out the summation in (2.1.10) and (2.1.13). One obtains

for p- shell nuclei for the second term in (2.1.10).

$$\sum_{\alpha,\beta} \langle \phi_{\alpha\beta} | e^{i\vec{q}\cdot\vec{r}_1} h(r_{12}) | \phi_{\alpha\beta} \rangle$$

$$= 12 \exp(-q^2 a_s^2/8) \times \langle oo || h(r) J_o(\tfrac{1}{2} qr) || oo \rangle$$

$$+ 8A_p \left[F^o - \frac{1}{12} G^1 \right]$$

$$+ A_p(A_p-1)\left[\frac{1}{22} \left(\langle oo|h(r)J_o(\tfrac{1}{2} qr)|| oo\rangle \langle 10|| J_o(qR)|| 10\rangle \right.\right.$$

$$+ \langle 10|| h(r) J_o(\tfrac{1}{2} qr)|| 10\rangle \langle oo|| J_o(qR)|| oo \rangle \Big)$$

$$- \frac{1}{11} \langle 10|| h(r) J_o(\tfrac{1}{2} qr)|| oo \rangle \langle oo|| J_o(qR)|| 10 \rangle +$$

$$+ \frac{5}{11} \left(\langle 01|| h(r) J_o(\tfrac{1}{2} qr)|| 01\rangle \langle 01|| J_o(qR)|| 01 \rangle \right.$$

$$- \langle 01|| h(r) J_2(\tfrac{1}{2} qr)|| 01 \rangle \langle 01|| J_2(qR)|| 01 \rangle \Big)$$

$$+ \frac{5}{22} \left(\langle oo|| h(r) J_o(\tfrac{1}{2} qr)|| oo\rangle \langle 02|| J_o(qR)|| 02 \rangle \right.$$

$$+ \langle 02|| h(r) J_o(\tfrac{1}{2} qr)|| 02 \rangle \langle oo|| J_o(qR)|| oo \rangle \Big)$$

$$- \frac{5}{11} \langle 02|| h(r) J_2(\tfrac{1}{2} qr)|| oo\rangle \langle oo|| J_2(qR)|| 02 \rangle \Big] \ldots (2.5.7)$$

Similarly for the third term we obtain

$$\sum_{\alpha\beta} \langle \alpha | e^{i\vec{q}\cdot\vec{r}_1} | \alpha \rangle \langle \phi_{\alpha\beta} | h(r) | \phi_{\alpha\beta} \rangle$$

$$= 12 \langle 00 \| e^{i\vec{q}\cdot\vec{r}_1} \| 00 \rangle \langle 00 \| h(r) \| 00 \rangle$$

$$+ 4A_p \left[F^0 - \frac{1}{12} G^1 \right] \left[\exp\left(-\frac{q^2 a_p^2}{4}\right) + \frac{6-q^2 a_p^2}{6} \exp\left(-\frac{q^2 a_p^2}{4}\right) \right]$$

$$+ A_p (A_p - 1) \frac{6-q^2 a_p^2}{6} \exp\left(-\frac{q^2 a_p^2}{4}\right) \left[\frac{3}{11} \langle 00 \| h(r) \| 00 \rangle \right.$$

$$\left. + \frac{1}{22} \langle 10 \| h(r) \| 10 \rangle + \frac{5}{11} \langle 01 \| h(r) \| 01 \rangle + \frac{5}{22} \langle 02 \| h(r) \| 02 \rangle \right.$$

... (2.5.8)

The various radial matrix elements of $h(r)$ and $J_1(qr)$ $h(r)$ appearing in (2.5.7) and (2.5.8) can be evaluated by the technique of section 4. We have carried out the evaluation upto six moments. The results are as follows :

$$(1) \langle 00 \| h(r) J_0(\tfrac{1}{2} qr) \| 00 \rangle = \frac{V_2}{\sqrt{\pi}\, a^3} \left[I_1 - \left(\frac{1}{2a^2} + \frac{q^2}{24}\right) I_2 \right.$$

$$+ \left(\frac{1}{8a^4} + \frac{q^2}{48a^2} + \frac{q^4}{1920}\right) I_3 - \left(\frac{1}{48a^6} + \frac{q^2}{192a^4} + \frac{q^4}{a^2} + \frac{q^6}{322560}\right) I_4$$

$$+ \left(\frac{1}{384a^8} + \frac{q^2}{1152a^6} + \frac{q^4}{15360a^4} + \frac{q^6}{645120a^2} + \frac{q^8}{92897280}\right) I_5$$

$$- \left(\frac{1}{3840a^{10}} + \frac{q^2}{9216a^8} + \frac{q^4}{92160a^6} + \frac{q^6}{2580480a^4} + \frac{q^8}{185794560a^2}\right) +$$

$$\left. + \frac{q^{10}}{40874803200} \right) I_6 \Big] . \quad \text{... (2.5.9a)}$$

$$(ii)\ \langle 10|h(r)J_0(\tfrac{1}{2}qr)||00\rangle = \frac{2}{\sqrt{3}\,\pi^{1/2}a^3}\Big[\frac{3}{2}I_1-\Big(\frac{5}{4a^2}+\frac{q^2}{16}\Big)I_2$$

$$+\Big(\frac{7}{16a^4}+\frac{5q^2}{96a^2}+\frac{q^4}{1280}\Big)I_3-\Big(\frac{3}{32a^6}+\frac{7q^2}{384a^4}+\frac{q^4}{1536a^2}+\frac{q^6}{215040}\Big)I_4$$

$$+\Big(\frac{11}{768a^8}+\frac{q^2}{256a^6}+\frac{7q^4}{30720a^4}+\frac{q^6}{258048a^2}+\frac{q^8}{61931520}\Big)I_5$$

$$-\Big(\frac{13}{7680a^{10}}+\frac{11q^2}{18432a^8}+\frac{q^4}{20480a^6}+\frac{q^6}{737280a^4}+\frac{q^8}{74317824a^2}+\frac{q^{10}}{2724986800}\Big)I_6-$$

... (2.5.9b)

$$(iii)\langle 10|h(r)J_0(\tfrac{1}{2}qr)||10\rangle = \frac{2\sqrt{2}}{3\sqrt{\pi}a^3}\Big[\frac{9}{4}I_1-\Big(\frac{21}{8a^2}+\frac{3q^2}{32}\Big)I_2$$

$$+\Big(\frac{41}{32a^4}+\frac{7q^2}{64a^2}+\frac{3q^4}{2560}\Big)I_3-\Big(\frac{23}{64a^6}+\frac{41q^2}{768a^4}+\frac{7q^4}{1280a^2}+\frac{q^6}{143360}\Big)I_4$$

$$+\Big(\frac{35}{512a^8}+\frac{23q^2}{1536a^6}+\frac{41q^4}{61440a^4}+\frac{q^6}{122880a^2}+\frac{q^8}{41287690}\Big)I_5$$

$$-\Big(\frac{23}{1536a^{10}}+\frac{35q^2}{12288a^8}+\frac{23a^4}{122880a^6}+\frac{41q^6}{10321920a^4}+\frac{11q^8}{4954452160a^2}+\frac{q^{10}}{1816657920}\Big)I_6$$

... (2.5.9c)

$$(iv)\ \langle 01|h(r)J_0(\tfrac{1}{2}qr)||01\rangle = \frac{\sqrt{2}}{3\sqrt{\pi}\,a^5}\Big[I_2-\Big(\ldots+\frac{q^2}{24}\Big)I_3$$

$$+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_4-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_5$$

$$+\left(\frac{1}{384a^8}+\frac{q^2}{1152a^6}+\frac{q^4}{15360a^4}+\frac{q^6}{645120a^2}+\frac{q^8}{92897280}\right)I_6\Big]\ldots(2.5.9d)$$

$$(v)\ \langle 02\|h(r)J_0\left(\tfrac{1}{2}qr\right)\|02\rangle=\frac{\sqrt{2}}{15\sqrt{\pi}a^7}\Big[I_3-\left(\frac{1}{2a^2}+\frac{q^2}{24}\right)I_4$$

$$+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_5-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_6\Big]$$

$$\ldots \qquad (2.5.9e)$$

$$(vi)\ \langle 01\|h(r)J_2\left(\tfrac{1}{2}qr\right)\|01\rangle=\frac{q^6}{6\sqrt{2\pi}\,a^5}\Big[\frac{I_3}{15}-\left(\frac{1}{30a^2}+\frac{q^2}{840}\right)I_4$$

$$+\left(\frac{1}{120a^4}+\frac{q^2}{1680a^2}+\frac{q^4}{120960}\right)I_5$$

$$-\left(\frac{1}{720a^6}+\frac{q^2}{6720a^4}+\frac{q^4}{241920a^2}+\frac{q^6}{31933440}\right)I_6\Big]\ldots(2.5.9f)$$

$$(vii)\ \langle 02\|h(r)J_2\left(\tfrac{1}{2}qr\right)\|00\rangle=\frac{3}{\sqrt{15}}\langle 01\|h(r)J_0\left(\tfrac{1}{2}qr\right)\|01\rangle$$

$$\ldots \qquad (2.5.9g)$$

The matrix elements F^0, F^1 are evaluated to give

$$F^0=\frac{a_s^2}{a_s^2+a_p^2}\left(-\frac{q^2\bar{a}^2}{12}\right)\exp\left(-\frac{q^2\bar{a}^2}{8}\right)\frac{\sqrt{2}}{\bar{a}^3\pi^{1/2}}\Big[I_1-\left(\frac{1}{2a^2}+\frac{q^2}{24}\right)I_2$$

$$+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_3-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_4$$

$$+\left(\frac{1}{384a^8}+\frac{q^2}{1152a^6}+\frac{q^4}{15360a^4}+\frac{q^6}{645120a^2}+\frac{q^8}{92897280}\right)I_5$$

$$-\left(\frac{1}{3840a^{10}}+\frac{q^2}{9216a^8}+\frac{q^4}{92160a^6}+\frac{q^6}{2580480a^4}+\frac{q^8}{185794560a^2}+\frac{q^{10}}{40874803200}\right)I_6\Big]$$

$$+\frac{a_p^2}{a_s^2+a_p^2}\exp\left(-\frac{q^2a^2}{8}\right)\frac{\sqrt{2}}{3a^3\pi^{1/2}}\Big[I_2-\left(\frac{1}{2a^2}+\frac{q^2}{24}\right)I_3$$

$$+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_4-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_5$$

$$+\left(\frac{1}{384a^8}+\frac{q^2}{1152a^6}+\frac{q^4}{15360a^4}+\frac{q^6}{645120a^2}+\frac{q^8}{92897280}\right)I_6\Big].$$

... (2.5.10)

$$\bar{\sigma}^1=\frac{12a_s^2}{(a_s^2+a_p^2)^{5/2}\pi^{1/2}}\exp\left(-\frac{q^2a^2}{8}\right)\Big[\left(1-\frac{q^2a^2}{12}\right)\left(I_1-\left(\frac{1}{2a^2}+\frac{q^2}{24}\right)I_2\right.$$

$$+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_3-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_4\Big)$$

$$+\left(\frac{1}{384a^8}+\frac{q^2}{115a^6}+\frac{q^4}{15360a^4}+\frac{q^6}{645120a^2}+\frac{q^8}{92897280}\right)I_5$$

$$-\left(\frac{1}{3840a^{10}}+\frac{q^2}{9216a^8}+\frac{q^4}{92160a^6}+\frac{q^6}{2580480a^4}+\frac{q^8}{185794560a^2}+\right.$$

$$\left.+\frac{q^{10}}{40874803200}\right)I_6$$

$$-\frac{1}{3a^2}\left[I_2-\left(\frac{1}{2a^2}+\frac{q^2}{24}\right)I_3+\left(\frac{1}{8a^4}+\frac{q^2}{48a^2}+\frac{q^4}{1920}\right)I_4\right.$$

$$-\left(\frac{1}{48a^6}+\frac{q^2}{192a^4}+\frac{q^4}{3840a^2}+\frac{q^6}{322560}\right)I_5$$

$$\left.+\left(\frac{1}{384a^8}+\frac{q^2}{1152a^6}+\frac{q^4}{15360a^4}+\frac{q^6}{645120a^2}+\frac{q^8}{92897280}\right)I_6\right]$$

... (2.5.11)

The matrix elements of h(r) are

$$(1)\ \langle 00\|h(r)\|00\rangle=\frac{Y2}{\sqrt{\pi}a^3}\left[I_1-\frac{I_2}{2a^2}+\frac{I_3}{8a^4}-\frac{I_4}{48a^6}+\right.$$

$$\left.+\frac{I_5}{384a^8}-\frac{I_6}{3840a^{10}}\right]\ldots \quad (2.5.12)$$

$$(11)\ \langle 10\|h(r)\|10\rangle=\frac{2Y2}{3\sqrt{\pi}\,a^3}\left[\frac{9}{4}I_1-\frac{21}{8a^2}I_2+\frac{41}{32a^4}I_3-\frac{23}{64a^6}\right.$$

$$\left.I_4+\frac{35}{512a^8}I_5-\frac{23}{1536a^{10}}I_6\right]\ldots \quad (2.5.13)$$

$$(111)\ \langle 01 \| h(r) \| 01 \rangle = \frac{\sqrt{2}}{3\sqrt{\pi}a^3}\left[I_2 - \frac{I_3}{2a^2} + \frac{I_4}{8a^4} - \frac{I_5}{48a^6} + \frac{I_6}{384a^8}\right]$$

... (2.514)

$$(1v)\ \langle 10 \| h(r) \| 10 \rangle = \frac{2}{\sqrt{5\pi}\ a^3}\left[\frac{3}{2} I_1 - \frac{3}{4a^2} I_2 + \frac{3}{16a^4} I_3 - \frac{3}{32a^6}\right.$$

$$\left. I_4 + \frac{11}{768a^8} I_5 - \frac{13}{7680a^{10}} I_6\right] \quad \ldots \quad (2.5.15)$$

$$(v)\ \langle 02 \| h(r) \| 02 \rangle = \frac{\sqrt{2}}{15\sqrt{\pi}a^7}\left[I_3 - \frac{I_4}{2a^2} + \frac{I_5}{8a^4} - \frac{I_6}{48a^6}\right]$$

... (2.5.16)

The F^0 and $\bar{G}^1$ are

$$F^0 = \frac{a_s^2}{a_s^2+a_p^2}\ \frac{\sqrt{2}}{\bar{a}^3\pi^{1/2}}\left[I_1 - \frac{I_2}{2\bar{a}^2} + \frac{I_3}{8\bar{a}^4} - \frac{I_4}{48\bar{a}^6} + \frac{I_5}{384\bar{a}^8} - \frac{I_6}{3840\bar{a}^{10}}\right]$$

$$+ \frac{a_p^2}{a_s^2+a_p^2}\ \frac{\sqrt{2}}{3\bar{a}^3\pi^{1/2}}\left[I_2 - \frac{I_3}{2\bar{a}^2} + \frac{I_4}{8\bar{a}^4} - \frac{I_5}{48\bar{a}^6} + \frac{I_6}{384\bar{a}^8}\right]$$

... (2.5.17)

$$\bar{G}^1 = \frac{12a_s^2}{(a_s^2+a_p^2)^{1/2}\pi^{1/2}}\left[\left(I_1 - \frac{I_2}{2\bar{a}^2} + \frac{I_3}{8\bar{a}^4} - \frac{I_4}{48\bar{a}^6} + \frac{I_5}{384\bar{a}^8} - \frac{I_6}{3840\bar{a}^{10}}\right)\right.$$

$$\left. - \frac{1}{3\bar{a}^2}\left(I_2 - \frac{I_3}{2\bar{a}^2} + \frac{I_4}{8\bar{a}^4} - \frac{I_5}{48\bar{a}^6} + \frac{I_6}{384\bar{a}^8}\right)\right] \ldots (2.5.18)$$

We shall also need the matrix elements $\langle n0 \| h(r) \| 0 \rangle$, (3.2.5). They can be easily evaluated to give

$$\langle n0 \| h(r) \| 00 \rangle = \left[\frac{\sqrt{(n+1)}\sqrt{(n+3/2)}}{\pi^{1/2}\, a^5} \right]^{1/2} \sum_{m=0}^{n} \frac{(-)^m}{(2a^2)^m} \frac{1}{\Gamma(m+1)\Gamma(n-m+1)\Gamma(m+3/2)} \sum_{y=0}^{\infty} (-)^y \left(\frac{1}{2a^2}\right)^y \frac{1}{y!} I_{(m+y+1)} \quad \ldots \quad (2.5.19)$$

where $I_{(m+y+1)}$ is the $(m+y+1)^{th}$ moment of $h(r)$.

The series in (2.5.19) is to be truncated at the desired moment.

Finally we may need matrix elements corresponding to the two particle centre of mass coordinates. For harmonic oscillator wavefunctions they can be easily evaluated. The results are as follows :

$$(i) \quad \langle 00 | J_0(qR) \| 00 \rangle = \exp(-q^2a^2/8) \quad \ldots \quad (2.5.20a)$$

$$(ii) \quad \langle 10 \| J_0(qR) \| 10 \rangle = \left(1 - \frac{q^2a^2}{6} + \frac{q^4a^4}{96} - \ldots\right) \exp\left(-\frac{q^2a^2}{8}\right) \quad \ldots \quad (2.5.20b)$$

$$(iii) \quad \langle 10 \| J_0(qR) \| 0 \rangle = \frac{q^2a^2}{4\sqrt{6}} \exp\left(-\frac{q^2a^2}{3}\right) \ldots (2.5.20c)$$

$$(iv)\ \langle 01 \| J_c(qR) \| 01 \rangle = \left(1 - \frac{q^2 a^2}{12}\right) \exp\left(-\frac{q^2 a^2}{3}\right) \ldots (2.5.20d)$$

$$(v)\ \langle 02 \| J_0(qR) \| 02 \rangle = \left(1 - \frac{q^2 a^2}{6} + \frac{q^4 a^4}{240}\right) \exp\left(-\frac{q^2 a^2}{8}\right)$$

$$\ldots (2.5.20e)$$

$$(vi)\ \langle 01 \| J_2(qR) \| 01 \rangle = \frac{q^2 a^2}{12} \exp\left(-\frac{q^2 a^2}{8}\right) \ldots (2.5.20f)$$

$$(vii)\ \langle 02 \| J_2(qR) \| 00 \rangle = \frac{q^2 a^2}{4\sqrt{15}} \exp\left(-\frac{q^2 a^2}{8}\right) \ldots (2.5.20g)$$

....

CHAPTER-III

In this chapter we present the results of numerical calculations for the nuclei ^4He, ^{16}O and ^6Li. The calculations are carried out in the framework developed in the preceding chapters and are described in section (1). In section (2) we discuss the implications of the results obtained in section (1). Section (3) is devoted to conclusions.

1. RESULTS OF NUMERICAL CALCULATIONS.

First of all we have examined the flexibility of our correlation factor (2.3.6). This is done for ^4He. In fig.(2) the charge form factor of ^4He is plotted as a function of q^2 as calculated by invoking Gaussian[31] ($-\exp(-r^2/b^2)$) correlation factor represented by solid curve for b = 0.95 fm. The dashed curve represents the calculation employing oscillatory ($-J_0(q_0 r)$) correlation factor[7] for $q_0 = 2.4 fm^{-1}$. The oscillator range parameter is taken 1.21 fm in both the cases. We then tried to reproduce the two curves of fig.(2) by invoking the proposed correlation factor (2.3.6). It was found that for $I_1 = -0.333\ fm^3$, $I_2 = 0.301\ fm^5$ and $I_3 = -0.232\ fm^7$ the

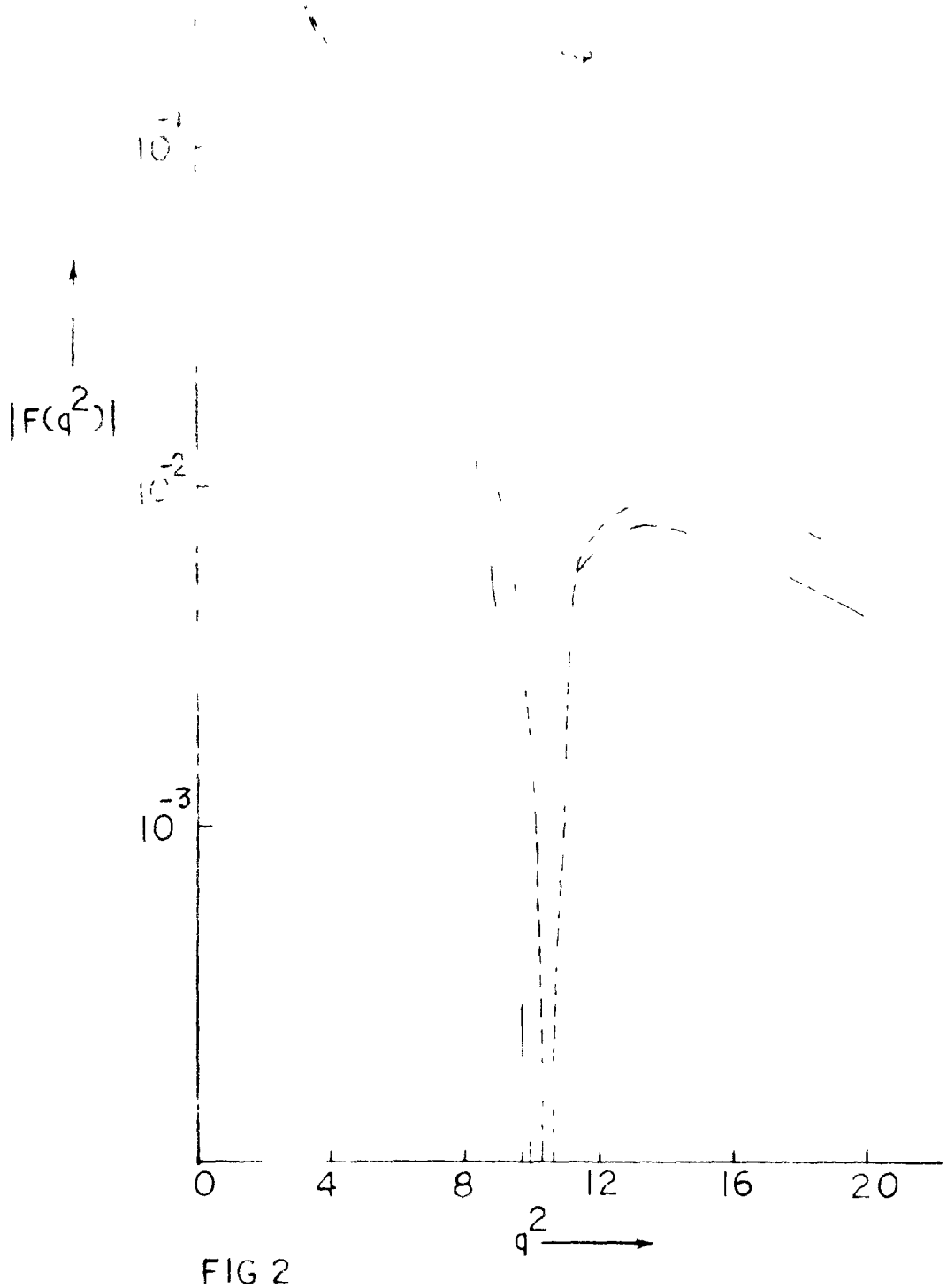

FIG 2

charge form factor calculated using (2.3.6) completely coincide with the solid curve. The agreement between the two calculations was close to three significant digits. Similar results were obtained in the case of the dashed curve also. Here also the overlap of the charge form factor curve calculated using (2.3.6) was almost complete with the dashed curve for

$$I_1 = I_2 = I_3 = 0 ,$$

and $I_4 = 15.873\ fm^9$, $I_5 = 142.546\ fm^{11}$, $I_6 = 447.133\ fm^{13}$. It may be mentioned that all the moments of the oscillatory correlation factor are zero and in fact it may be considered as a long range correlation factor. Therefore, while seeking agreement of the charge form factor curves calculated using (2.3.6) and the oscillatory correlation factor one has to make a first few moments of (2.3.6) zero and carry out the calculation with a few higher moments. The agreement between the charge form factor curves was brought about by a least square fit. The above agreements show the remarkable flexibility of the proposed correlation factor (2.3.6) in assimilating two entirely different correlation factors. Because of this flexibility we have been able to obtain precision

fitting of the charge form factor data of ^4He, ^{16}O and particularly of ^6Li for which it is real hard to get an adequate fit[49].

Fig.(3) gives the fits of the ^4He charge form factor data for the oscillator range parameter 'a' ranging from 1.2 to 1.5 fm . The finite size of the proton was taken into account by using a realistic form factor[50]

$$f_p(q^2) = 0.5\left(\frac{2.5}{1+q^2/15.7} - \frac{1.6}{1+q^2/26.7} + \frac{1.16}{1+q^2/8.19}\right) - 0.05$$

The charge form factor data was taken from Sick et.al[27] which when combined with the older data[27] gives a total of 86 data points. In all the cases the fits are excellent and all the curves for different values of 'a' lie within the crosshatched area shown in fig.(3). However, it was found that to obtain a good fit for lower values of 'a' the expansion (2.3.6) had to be taken up to a large number of moments. The results for a = 1.20 fm are given in table I. This indicates that for lower values of 'a' the correlation factor is not short ranged as one would expect. For a >1.30fm excellent fits of the data were easily obtained by taking only the first three

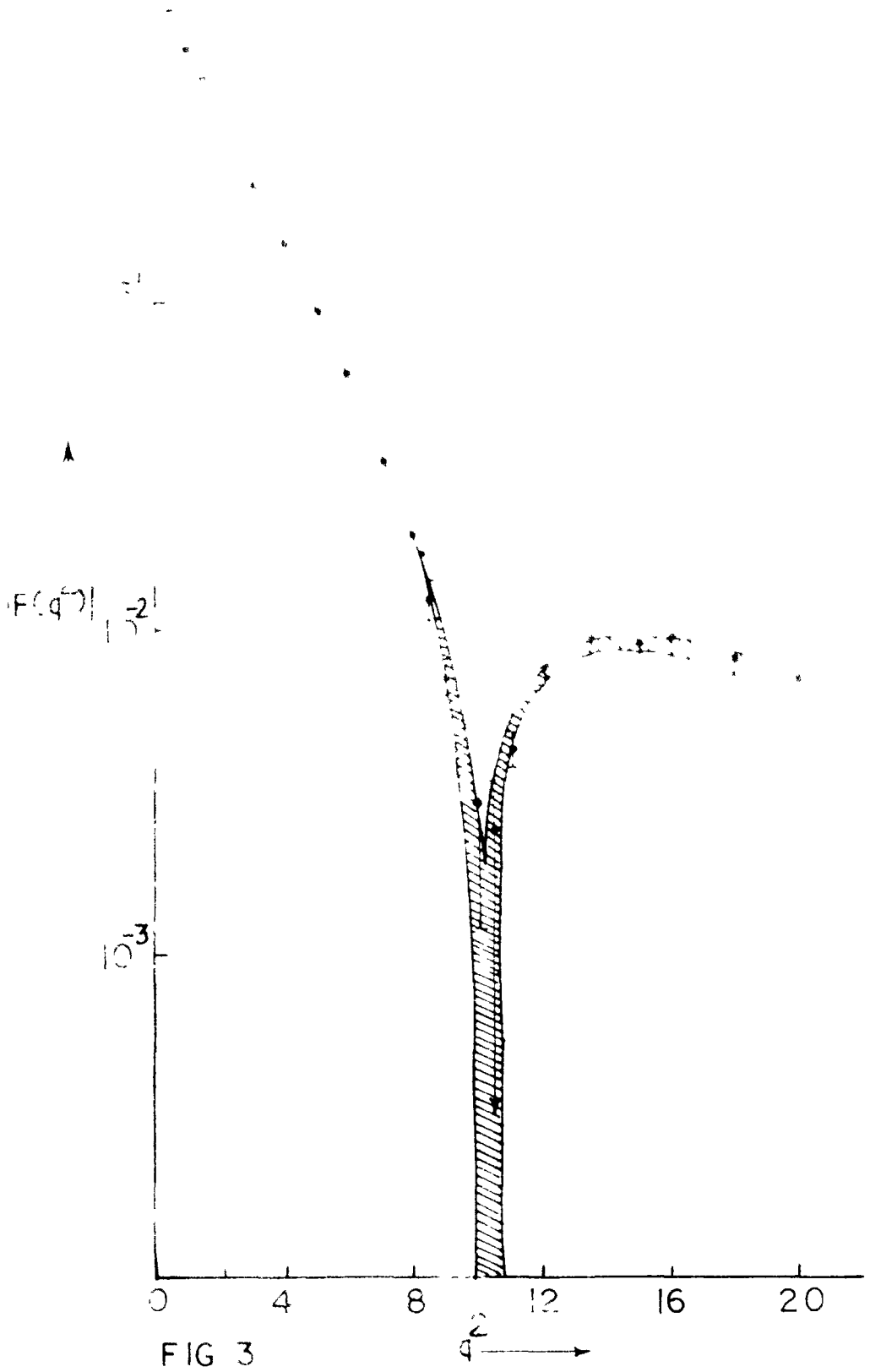

FIG 3

TABLE I

I_1	I_2	I_3	I_4	I_5	I_6	Chi Square
-0.476	-0.594	-0.591	0.000	0.000	0.000	340
-0.572	-0.790	-0.597	1.070	0.000	0.000	263
-0.847	-1.270	-0.538	1.825	-0.974	0.000	136
-0.915	-1.370	-0.460	-2.381	-73.525	-27.286	126

Results of the fitting of charge form factor data of ^4He for a = 1.20fm . It is seen that the Chi square decreases slowly as the number of correlation parameters (moments) is increased.

TABLE II

a	$I_1(fm^3)$	$I_2(fm^5)$	$I_3(fm^7)$
1.38	0.050	-0.354	-1.927
1.50	0.816	1.079	-1.059
1.60	1.465	2.266	-1.055

Results of the fitting of charge form factor data of ^4He for other values of 'a'.

moments as parameters. The results are given in table II for a = 1.38, 1.50 and 1.60 fm.

Next, we have generated the $F_0 + F_{1p-1h}$ and F_{2p-2h} contributions according to (2.2.6), (2.2.7) and (2.2.8). This was done by taking the excitations upto the oscillator quanta n = 20. Further increase in n does not make any change in the particle-hole excitation contributions. The results are displayed in fig.(4) for a = 1.38, 1.50 and 1.60 fm. It is seen that F_{2p-2h} contributions are quite dominant in the high momentum transfer region. Further, in all the cases we find that to a good degree $F_0 + F_{1p-1h}$ and F_{2p-2h} contributions are independent of 'a' which implies that they are independent of the single particle orbitals employed. This may be taken as a sufficient proof that the elastic electron scattering is sensitive to correlations and can distinguish between the shell model and the correlation aspects of the total nuclear wavefunction. An approximate particle-hole separation for the various contributions in ^{4}He in Brueckner as well as Jastrow theory were carried out by Degli Atti and Kallio[39]. Our results are in sharp contrast to that of the above reference where no consideration

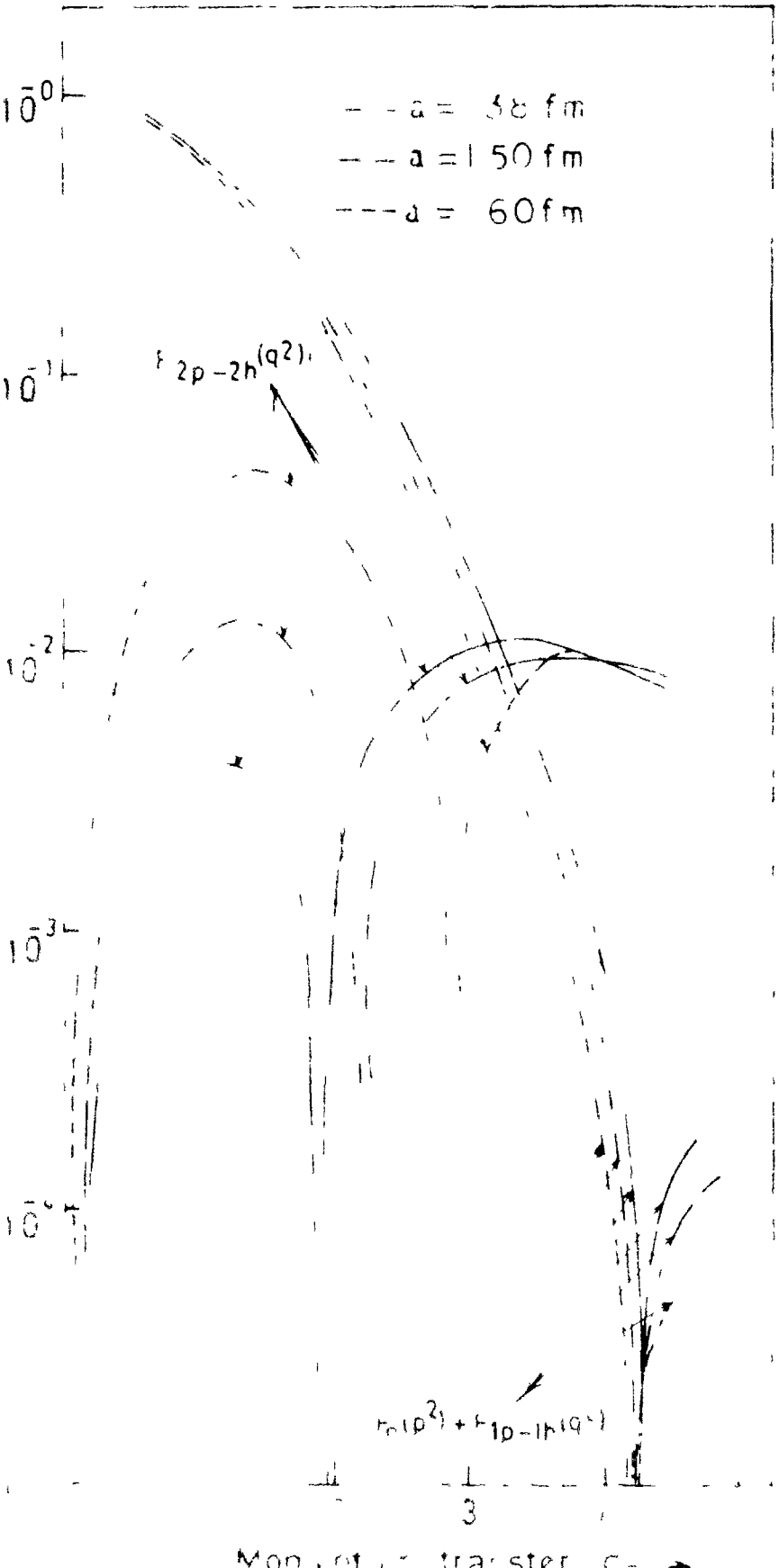
a = 56 fm
a = 1.50 fm
a = 60 fm
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$F_{2p-2h}(q^2)$
$F_{1p-1h}(q^2) + F_{1p-1h}(q^2)$
3
Momentum transfer q →

FIG4.

of the centre of mass motion was made while carrying out the particle-hole separation. In Degli Atti and Kallio's work the 2p-2h contributions were quite small which lend support to , inadvertently, that a major part of the experimental charge form factor curve could be described with a shell model picture of ^{4}He. Our results also show that correlations (2p-2h) ex- tion contributions) are independent of the single particle orbitals employed. However, some dependence on 'a' for the various contributions may be seen in fig.(4). The reasons for this dependence shall be discussed in the next section.

In figs.(5) and (6) we have made a comparison of the exact and approximate separations of the contributions due to particle-hole excitations in the charge form factor for a =1.38 and 1.60 fm respectively. The approximate separation of contributions was carried out according to the prescription given in section 2 of chapter II. It may be seen that there is remarkable agreement in the various contributions in the regions of momentum transfers where these contributions are dominant. The reasons for this agreement were discussed in detail in section 2 of chapter II. The above agreement gives us confidence in applying our approximate prescription for separation of particle-hole

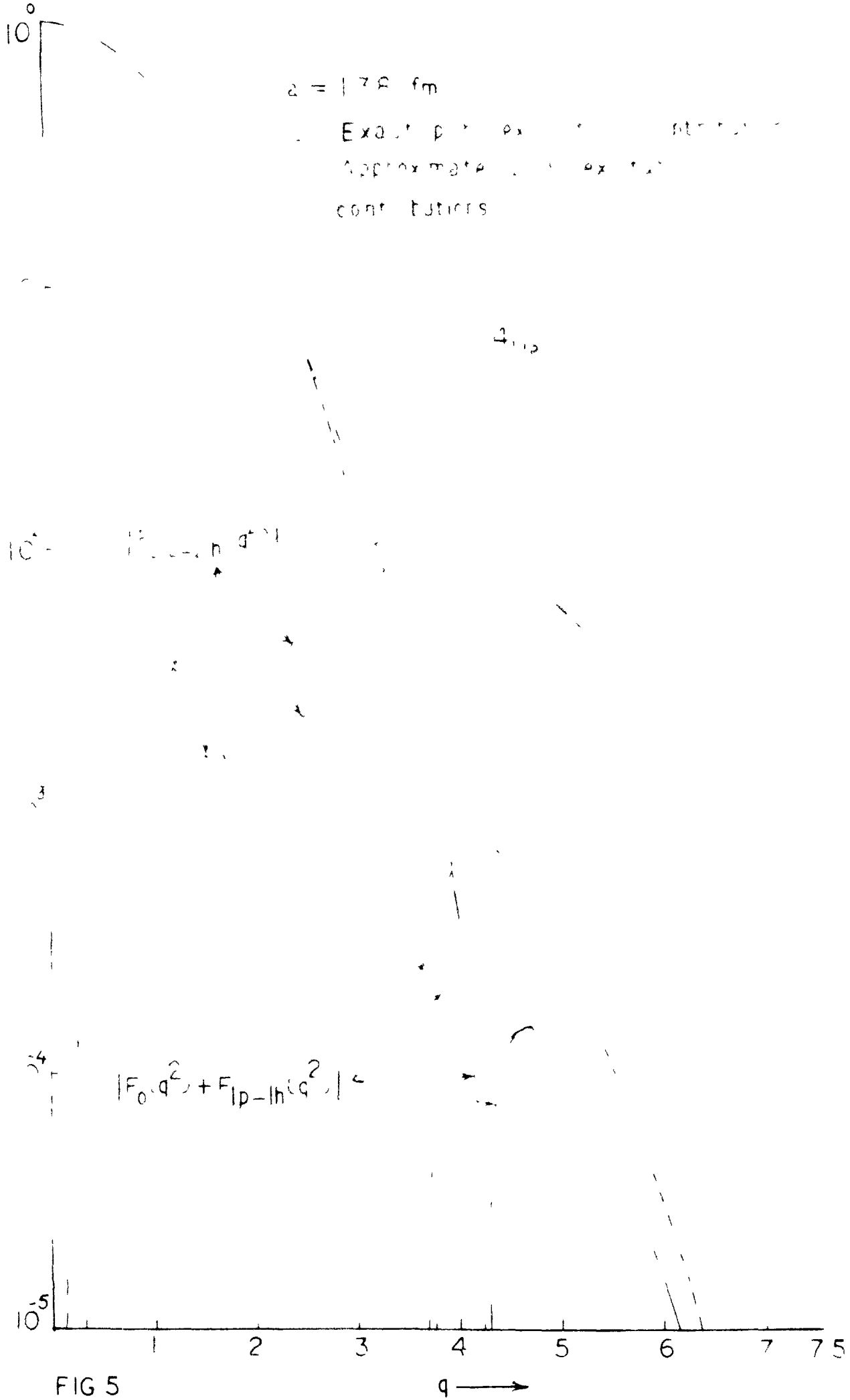
10°
a = 1.78 fm
Exact
Approximate
contributions
$|F_0(q^2) + F_{1p-1h}(q^2)|^2$
10^{-5}
1
2
3
4
5
6
7
7.5
q

FIG 5

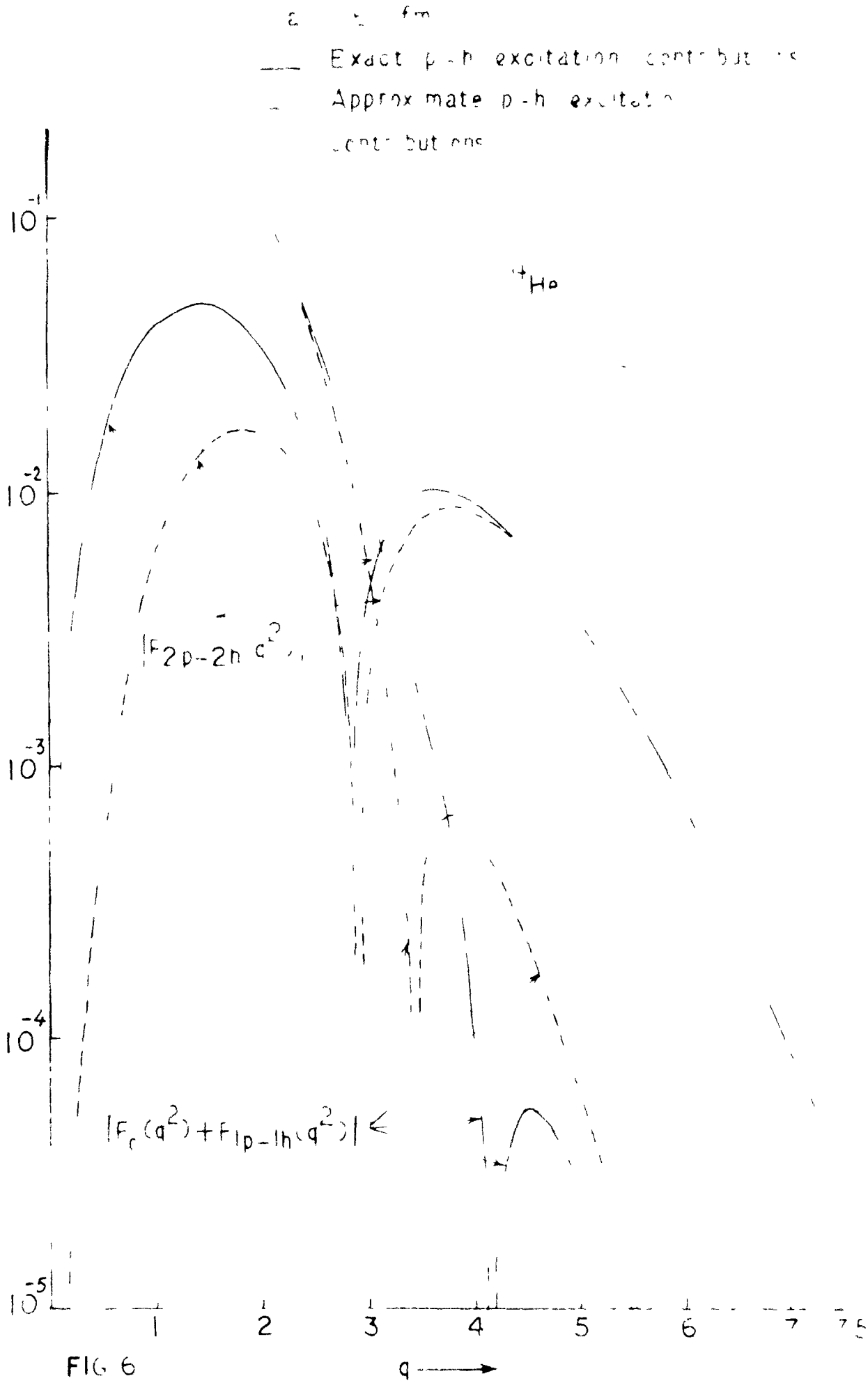
fm
Exact p-h excitation contributions
Approximate p-h excitation contributions
^{4}He
$|F_{2p-2h}(q^2)|$
$|F_c(q^2)+F_{1p-1h}(q^2)|$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
2
3
4
5
6
7
7.5
q

FIG 6

contributions in other nuclei.

In figs. (7) and (8) we have obtained fits of the charge form factor data [28,49] of ^{16}O and ^{6}Li. For these two nuclei the correlations were taken to be effective only in the relative s state. Taking correlations in higher partial waves did not make any difference as the contributions due to these partial waves were found to be quite small. The following oscillator range parameters were used :

^{16}O : $a_s = a_p = 1.766$ fm

^{6}Li : $a_s = 1.616$, $a_p = 2.060$ fm ,

where a_s and a_p are the oscillator parameters for the s and p shell nucleons respectively.

The following values of the correlation parameters (moments) were found to give the best fit :

^{16}O : $I_1 = 0.239$ fm^3 , $I_2 = 0.150$ fm^5, $I_3 = -2.595$ fm^7 and $I_4 = 3.800$ fm^9.

^{6}Li : $I_1 = 0.147$ fm^3, $I_2 = 0.421$ fm^5, $I_3 = -0.955$ fm^7 and $I_4 = -2.329$ fm^9.

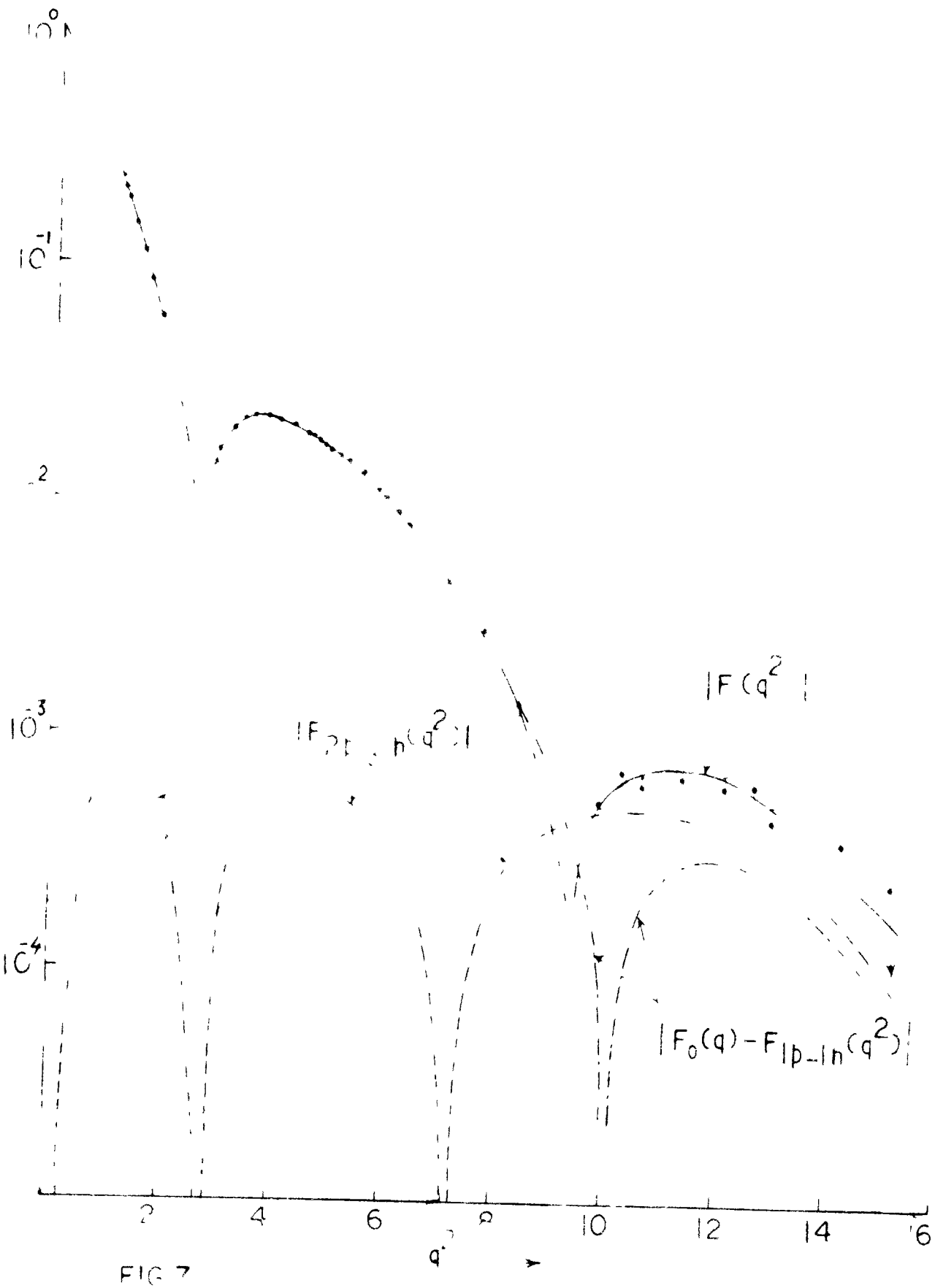

FIG 7

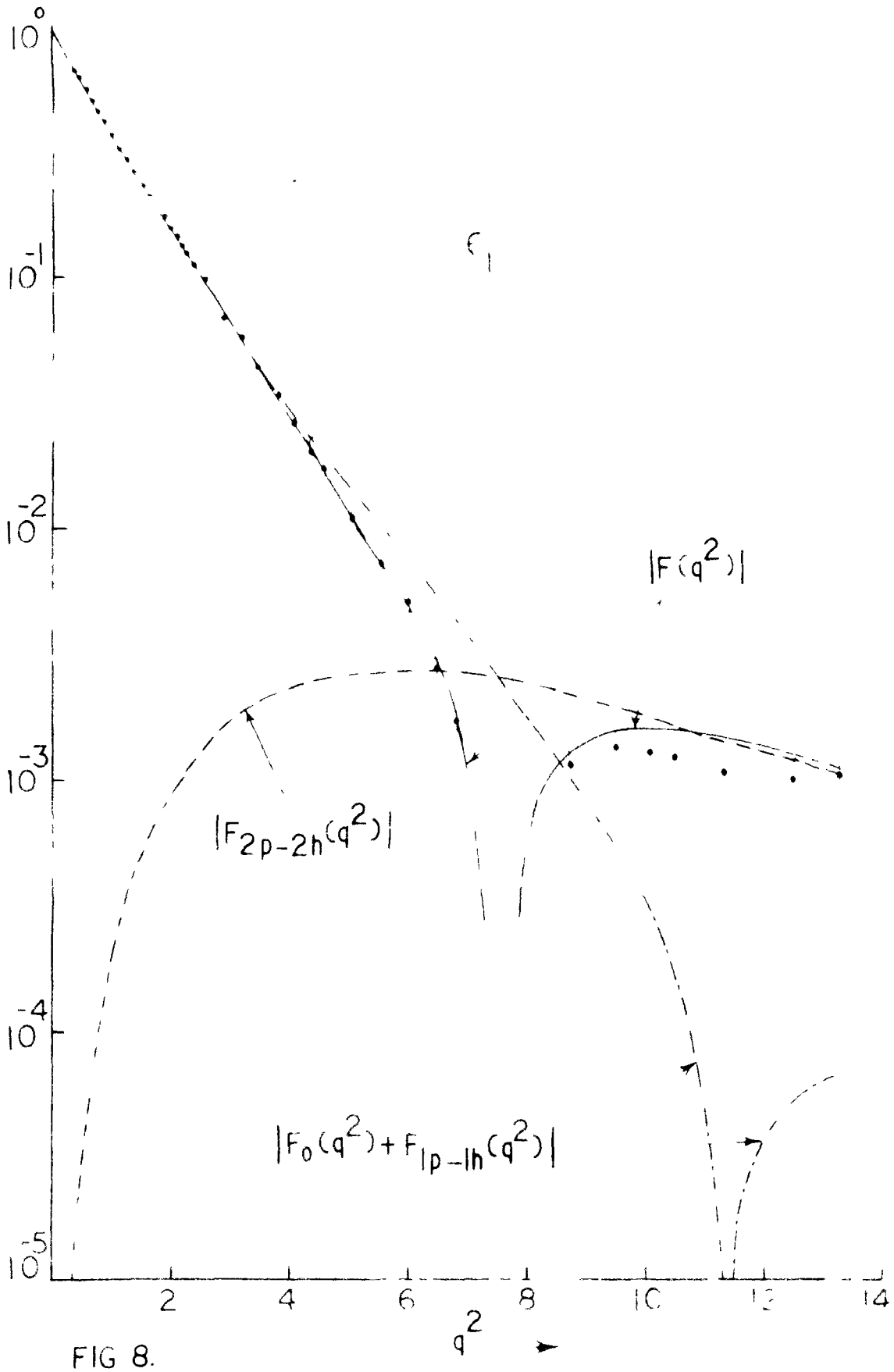

FIG 8.

It may be mentioned here that no adequate fit could be found in case of ^{6}Li for $a_s = a_p$. The solid curves in Figs.(7) and (8) represent the total form factor. The dashed curves represent the contribution due to 2p-2h excitations(F_{2p-2h}) and the dot-dashed curves represent the unperturbed plus the 1p-1h excitation ($F_o + F_{1p-1h}$), contributions. These contributions were separated out according to our approximate prescription.

2. DISCUSSION .

The dominance of F_{2p-2h} contribution in the high momentum transfer regions for all the three nuclei considered in the present study may be taken as a sufficient indication of the fact that elastic electron scattering is sensitive to correlations and can distinguish between the shell model and the correlation aspects of the total nuclear wavefunction. This result is in direct contradiction to the earlier findings[21,23] where it was claimed that for a given Jastrow wavefunction, it is possible to construct a new Slater determinant which gives a density very similar to the one calculated with the Jastrow wavefunction. Further, we have demonstrated for ^{4}He(Fig.4) that the correlation effects are independent of the single particle orbitals employed.

This is done by showing that the contributions $F_o + F_{1p-1h}$ and F_{2p-2h} are independent of the oscillator parameter 'a'. However, it may be seen that some dependence on 'a' occurs, particularly, in F_{2p-2h} contributions in the low momentum transfer region. Similarly, there is some dependence one 'a' in $F_o + F_{1p-1h}$ contribution in the high momentum transfer region around $q = 4.5\ fm^{-1}$. However, these small dependence on 'a' do not affect our conclusions because of the following reasons. In the low momentum transfer region the F_o+F_{1p-1h} contributions are dominant and to a good degree are independent of 'a'.Similarly, in the high momentum transfer region the F_{2p-2h} contributions are dominant and are independent of 'a'. Thus, only the smaller contributions in the low or high momentum transfer regions show dependence on 'a'. The above dependence on 'a' may arise because the smaller contributions are results of differences of two large but comparable quantities. For example, the F_{2p-2h} contribution is the difference of $F(q^2)$ and $F_o + F_{1p-1h}$ which are comparable in the low momentum transfer region. Therefore, a small difference in the fitting of the charge form factor data in the low momentum transfer region for different values of 'a' may give rise to large differences in the F_{2p-2h} contributions. There may be

another reason for the above differences. If we had performed the cluster expansion to all orders the above differences might have been reduced considerably. This follows from the fact that truncating the cluster expansion at some stage and then performing the fits with different values of 'a' leads to some extent to different total nuclear wavefunctions which in turn may lead to differences among the various particle-hole contributions. This may also be the reason for the differences of various contributions in the region of the minimum in the sharre form factor around $q \sim 5$ fm^{-1} where the neglect of higher order clusters may become serious[54]. This was also the reason for not carrying out the particle-hole separation for the fits obtained with a = 1.20 fm . It is evident from table (I) six correlation parameters (moments) were required to obtain an adequate fit which in turn implies that correlations for a = 1.20 fm would be long ranged and thus the higher order clusters in this case would be more important. For those values of 'a' for which the oscillator wavefunctions, in case of light nuclei, are close to the maximum overlap[48] or self consistent orbitals the correlations would be short ranged. In this situation the cluster expansion is expected to be

more convergent. Alternatively, we may choose those values of 'a' for which the charge form factor data at low momentum transfers are fitted nicely. For ^{4}He the desired value of 'a' was found to be 1.38 fm . The values of 'a' for ^{16}O and ^{40}Ca were chosen accordingly. However, we have not carried out the calculations with the clusters of higher order because of the following reasons:

(a) To separate out the particle-hole excitation contributions from the higher order clusters is extremely difficult.

(b) We intended to show that the conclusions of the earlier analysis[21,23] which are based upon the cluster expansion of the same order as in the present analysis are untenable.

Since, correlations manifest because of their interplay with the centre of mass correlations we may have some confidence in our approximate description for separating out the particle-hole contributions. This is also demonstrated in figs.(5) and (6), where we have compared the exact and the approximate separations. Thus, it may follow that the inclusion of higher order clusters may not appreciably change the F_0+F_{1p-1h}

contributions, though, now in addition to ρ_{2p-2h} contributions we may have some ρ_{3p-3h} contributions etc.. Moreover, the inclusion of higher order clusters may change the correlation parameters to some extent as some of the high momentum components in the nucleon wavefunction may be provided by the higher order clusters.

The reason why we have made such a detail study of ^{4}He nucleus is mainly because to authors knowledge it is the only nucleus where separation of the contribution due to particle hole excitations in Brueckner as well as Jastrow theory has been carried out[39]. Further, a good deal of controversy regarding the suitability of electron scattering in determining correlations has centered around this nucleus[35].

It may be seen in Fig.(4) that correlation effects (ρ_{2p-2h}) start becoming important after 1.5 fm^{-1}. This means that the charge form factor data for q less than 1.5 fm^{-1} may be adequately described with a shell model description of ^{4}He but for explaining the data at higher q values correlations are necessary. If we choose a heavier nucleus, it is obvious, since correlations manifest because of their interplay with

centre of mass correlations, that the shell model picture would be an adequate description upto some q [illegible] 1.5 fm^{-1}. For example in ^{6}Li the data could be described within the shell model scheme upto ~ 2.2 fm^{-1} whereas the corresponding value of the momentum transfer in ^{16}O is $\sim 2.9 fm^{-1}$ Thus, the ranges of momentum transfer in which the shell and correlation effects are important would be different in light and heavy nuclei. For more lighter systemslike A = 2 and A = 3, though for the former case the [illegible] data are not available, the values of momentum transfers upto which the shell model may be taken as a valid description would be much smaller. Therefore, it would be not adequate to describe these systems in terms of single particle orbitals. They form a separate class by themselves distinct from other nuclei. For these systems almost all the calculations have been carried out with correlated wavefunctions. But, here also the question arises whether a given one body density can uniquely determine the correlations.

One may be worried by the arbitrariness of ϕ and h(r) in this study , for example for ^{4}He. One may be [illegible] to

hold the opinion that the parameters of ϕ and h(r) should be picked up by minimising the energy using a realistic potential. The above arbitrariness arises out of the fact, as stated in section (4) of chapter I, that Jastrow correlation factor scatters particles into all states. In our view this arbitrariness is an inherent feature of finite systems. In a full variational (energy) calculation within the framework of Jastrow theory one will end up with some final result no matter what single particle orbitals one employes provided one uses a state dependent and extremely flexible h(r). However, it may be expected that starting with different sets of single particle orbitals one may obtain different rates of convergence in the energy in different orders of the cluster expansion. One may still define the set of single particle orbitals (for which the cluster expansion is convergent) by requiring that in the particle-hole expansion of the Jastrow wavefunction the 1p-1h excitation terms be minimum or one may define a set of maximum overlap orbitals and these would be independent of the original single particle orbitals employed. The correlations over the

new orbitals may be taken as a representation of the true correlations. It is because of this reason that we have collected the $F_0 + F_{1p-1h}$ contributions to the charge form factor which may be taken, to a large extent, as contribution
out of the new single particle orbitals mentioned above. Thus, we have been able to counter the earlier findings of Ripka et.al[23] and Fink et.al[21] that in Jastrow theoretic explanation of the charge form factor data the correlations depend entirely upon the single particle orbitals. Our conclusions, therefore, give credibility to the wavefunctions determined from elastic electron scattering. One may use these wavefunctions in some other processes which involve, for example, two body intrinsic operators. It would, therefore, serve not much purpose if ϕ and h(r) are determined variationally by choicing a relalistic potential. This would, in fact, be
...tagonistic to the whole approach presented in this study. It is well known that the variational wavefunctions do not generally account for the elastic electron scattering and the binding energy simultaneously. Also, in many other processes the use of the above variational wavefunctions have not been

very enlightening.

One may raise some objections about the choice of h(r),(2.3.6), that we have made in the present study. For example, it would seem that (2.3.6) is suitable for describing very low momentum properties of nuclear charge distributions, it seems inappropriate for probing the high momentum transfer regions necessary to see evidence for the short range correlations. Neither does it appear a suitable form for describing the " overshoot" usually encountered in the correlation factors due to intermediate range attraction in the NN interaction. However, it may be pointed out that the expansion (2.3.6) has been taken as the definition of the correlation factors with I_n as its parameters. The above objections would have been valid if we would have claimed that we were attempting to determine the precise values of the moments of h(r). In momentum space h(r) would be of the form $A + Bq^2 + Cq^4 + \ldots$, where A , B etc.contain I_1 , I_2 etc. respectively. Thus, the fitting of the charge form factor data with (2.3.6) corresponds to a polynomial determination of h(r) in momentum space. Since a polynomial is an extremely flexible function we have been able

to obtain precision fitting of the data for arbitrary values of the oscillator range parameter. In a fit of the above kind I_n cannot exactly correspond to the true moments of h(r). Truncating the expansion (2.3.6) after some I_n introduces a kind of non-locality in the correlation factor. However, if we assert that the parameters I_n are not very far from the true moments of h(r) the "overshoot" mentioned above is hiddem in I_n . This may be seen in the following way. Let us assume that h(r) is sufficiently short ranged. We now refer to table II. Since I_3 is negative for all the values of a it may be assumed that h(r) is negative for larger values of r as I_3 would be sensitive to the tail or exterior region of h(r). As r decreases h(r) would become positive since I_1 is positive. As r decreases further there would arise ambiguity in the behaviour of h(r) around r = 0 since the data in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1} cannot be expected to give information on h(r) around r = 0. Using relations (2.3.7) and (2.3.8) we have plotted h(r) for a = 1.38 fm. Fig.(9) shows h(r) for C = -1, 0,0.5 and 1.0. Though there could be infinite possibilities corresponding to different values of C and different functional

forms used in (2.3.7) but it is certain that h(r) changes sign at least once which amounts to " overshooting" mentioned above. For a = 1.20 it may be difficult to find the above overshoot from the correlation parameters. As stated earlier the correlation factor corresponding to the above value of a is considerably long ranged which may give rise to important higher order contributions. Therefore, not much reliance can be placed on correlation parameters obtained for a = 1.20 fm.

It may be seen from fig.(9) that the correlation factor is much more uncertain in the region defined by $0 \leq r \leq 0.6$ fm. It, thus, follows that electron scattering in the momentum transfer range $0 \leq q \leq .5$ is governed more by the behaviour of h(r) for $r \geq 0.6$ fm which corresponds to the intermediate and the tail region of the NN interaction. Thus, contrary to the assertions of earlier authors[19,43,44], the elastic electron scattering in the momentum transfer range $0 \leq q \leq 4.5$ is sensitive to the tail and the exterior region of the NN interaction.

Let us point out that the wavefunction of ^{4}He obtained by employing the so called realistic interactions yield a 10 %

mixture of D state due to the tensor force in the MN interaction[52]. This component of the wavefunction is particularly important for describing the charge form factor for large momentum transfers. The presence of a large D state probability may complicate the extraction of correlation parameters. However, with the presence of a D state mixture the main point of the present study is not disproved unless it is shown that the D state probability can be completely taken care of by single particle orbitals and at the same time diminishes very considerably the effect of 2p-2h excitations.

A D state mixture in ^{4}He may give rise to some deformation in this nucleus. A characteristic of the high energy angular distributions from the deformed nuclei is a smoother variation with angle than shown by the distribution from spherical nuclei. It is reasonable to attribute this behaviour to the averaging over angles of the deformed nuclear surface so that the radial distribution of charge which is effective in the scattering is more gradually tapered at the nuclear surface than in the case of spherically symmetric nuclei. Therefore, a small deformation in ^{4}He is likely to affect

the value of the oscillator range parameter and also may make some difference in the $P_0 + P_{1p-1h}$ contributions. However, the wavefunctions obtained in the present study may still be used in those processes which are not sensitive to nuclear deformation.

In this connection let us also note that precise measurement of 2p-2s transition energy in muonic ^4He[54] provides a most precise test of quantum electrodynamics which is verified within 0.25 % by assuming completely spherical distribution of charge[27,54]. An important correction in 2p-2s energy splitting stems from the polarization of ^4He core due to muon. It is, therefore, not unreasonable to expect that a 10 % D-state probability in ^4He wavefunction would greatly affect the credibility of quantum electrodynamics. Besides, it is yet to be decided whether the presence of D state in ^4He is an inherent feature or a consequence of inadequate description of the whole problem. It is, therefore, reasonable to conclude that description of ^4He with a large D state probability is a matter of theoretical prejudice unsubstantiated by any experimental evidence.

The case of the ^{12}C nucleus, which we have not studied, is a bit tricky. On the one hand this nucleus has been thought to be oblately deformed consistent with α - cluster triangle model[55]. On the other hand it was shown by Zofka and Rinka[55] that this nucleus could be equally well described by an RPA vibration of a spherical core instead of by a deformed intrinsic state. Because of these complications we have not included this nucleus in our study.

From the results of the three nuclei that we have considered here it may be concluded that the correlation effects in ^{40}Ca would be important in the experimentally unexplored momentum transfer region. For this reason and along with the fact that Born approximation will not be valid for such a heavy nucleus we have not studied it here.

In case of ^{6}Li we could not find an adequate fit for $a_c = a_p$. The following tentative reasons may be assigned for this finding :

(a) The nucleons in the p-shell may be loosely bound which would be consistent with alpha-deuteron model of ^{6}Li.

(b) The neglect of the higher order clusters may be responsible

for the large difference between a_s and a_p.

(c) The use of a state dependent correlation factor may considerably reduce the difference between a_s and a_p.

The above points need further investigation. An exhaustive review concerning the theoretical explanation of the charge form factor data of ^{6}Li is given in ref. 49).

It was shown in section (5) of chapter I that the Fourier components of h(r) with a wavelength smaller than $4\pi/q_{max}$ will not be determined by the charge form factor data. It may be also seen from the second term of (1.5.2) that if an electron transfers a momentum q to a single nucleon then this nucleon will have a relative momentum q/2 with respect to other nucleons at rest. Thus, it may be expected that the use of the wavefunctions obtained form the elastic electron scattering in processes involving the two body intrinsic operators will be appropriate roughly upto $q_{max}/2$.

Finally, it may be stated that the parameters of the correlation factors that we have determined here may not be very reliable as the contributions of relativistic effects and meson exchange currents have been completely neglected. But, for processes which involve the two body intrinsic operators

the interaction of correlation is also fraught with difficulties[33]. Nevertheless, our conclusion that a knowledge of one body density alone is sufficient to distinguish between the shell model and the correlation aspects of wavefunctions is not affected by the neglect of above effects.

3. CONCLUSION.

In the present study we have made an attempt to determine the dynamical short range correlations from elastic electron scattering data. Three nuclei, namely ^{4}He, ^{16}O and ^{6}Li have been considered. By making a particle-hole expansion of the Jastrow wavefunction it has been shown that the correlations manifest because of their intensity with the centre of mass correlations even for processes which involve one body intrinsic operators. It is argued that for light nuclei at large momentum transfers the particle-hole expansion, though mathematically exact, may lead to entirely wrong conclusions unless the effect of the centre of mass motion is taken into account

from the very beginning. Thus, our results and conclusions are in direct contradiction with the findings of earlier authors[21-23] who did not pay any attention to the importance of the centre of mass motion in interpreting the elastic electron scattering data.

However, it is shown that an unambiguous determination of correlations is not possible by analysing the charge form factor data in the momentum transfer range $0 \leq q \leq 4.5$ fm^{-1}. In this respect the recent measurement of Arnold et.al.[56] of the charge form factor data of ^{4}He upto 8 fm^{-1} would be extremely helpful. The calculations including the meson exchange and relativistic effects, which are not difficult to take into account, and employing the data of Arnold et.al.would give certainly more detailed and valuable information on correlation. Obviously, the data at higher momentum transfers for other nuclei would also be extremely useful. However, this may be mentioned that elastic electron scattering data do not distinguish between the correlations in singlet and triplet states. Only the average of the two may be determined.

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APPENDIX

Let us suppose that the nucleons are moving in a spherically symmetric potential, $V(r_1^2)$. The wavefunctions of the nucleons will be a product of the functions of r_1 etc. If this wavefunction can be written as the product or a finite sum of the products of functions of the centre of mass and intrinsic coordinates, the following relation must hold

$$V(r_1^2) + V(r_2^2) + \ldots + V(r_A^2) = U(\vec{\xi}_1, \vec{\xi}_2, \ldots \vec{\xi}_{A-1}) + W(\vec{R}) \qquad \ldots \quad (A1)$$

where

$$\vec{\xi}_i = \vec{r}_i - \vec{R}$$

and

$$\vec{R} = \sum_{i=1}^{A} \vec{r}_i / A .$$

Obviously,

$$\sum_{i=1}^{A} \vec{\xi}_i = 0$$

If we put in (A1)

$$\vec{\xi}_1 = \vec{\xi}_2 = \ldots = \vec{\xi}_A = 0$$

we obtain

$$W(\vec{R}) = AV(R^2) + \text{constant} \quad \ldots \quad (A2)$$

Similarly, if we put $\vec{R} = 0$ we get

$$U(\vec{\xi}_1, \vec{\xi}_2, \ldots \vec{\xi}_{A-1}) = V(\xi_1^2) + \ldots + V(\xi_A^2) + \text{constant}$$

$$\ldots \quad (A3)$$

substituting in (A1)

$$r_1^2 = r_2^2 = \ldots\ldots = r_A^2 \quad \ldots \quad (A4)$$

we obtain making use of (A2) and (A3)

$$AV(r_1^2) = V(\xi_1^2) + V(\xi_2^2) + \ldots + V(\xi_A^2) + AV(R^2) +$$

$$\text{constant} \quad \ldots \quad (A5)$$

Let us further put

$$\xi_1^2 = \xi_1^2 = \ldots = \xi_A^2 \quad \ldots \quad (A6)$$

Therefore, (A5) becomes

$$V(r_1^2) = V(\xi_1^2) + V(R^2) + \text{constant} \ldots \quad (A7)$$

condition (A4) together with (A6) implies

$$\sum_{i \neq 1} \cos \alpha_{11} = \sum_{i \neq 2} \cos \alpha_{21} = \ldots\ldots = \sum_{i \neq A} \cos \alpha_{A1}$$

$$\ldots \quad (A8)$$

where α_{mk} is the angle between $\vec{r}_m$ and $\vec{r}_k$.

Now R^2 with the condition (A 4) is

$$R^2 = \frac{r_1^2}{A} + \frac{r_1^2}{A^2} \sum_{i \neq j} \cos\alpha_{ij} .$$

Making use of (A 6)

$$R^2 = \frac{r_1^2}{A} \left(1 + \sum_{k=1,A}^{k \neq m} \cos\alpha_{mk}\right)$$

Similary

$$\xi_1^2 = r_1^2 + R^2 - 2\vec{r}_1 . \vec{R}$$

$$= r_1^2 + \frac{r_1^2}{A}\left(1 + \sum_{k=1,A}^{k \neq m} \cos\alpha_{mk}\right) - \frac{2r_1^2}{A} \sum_{k=1,A}^{k \neq m} \cos\alpha_{mk} - \frac{2r_1^2}{A}$$

$$= \frac{r_1^2}{A} (A - 1) - \sum_{k=1,A}^{k \neq m} \cos\alpha_{mk} .$$

Differentiating (A 7) with respect to α_{mn}

$$0 = -\frac{r_1^2}{A}\left[\frac{}{mn} \sum_{k=1,A}^{k \neq m} \cos\alpha_{mk}\right] \frac{\partial V(\xi_1^2)}{\partial \xi_1^2} +$$

$$+ \frac{r^2}{A}\left[\frac{\partial}{\partial \alpha_{mn}} \sum_{k=1,A}^{k \neq} \cos\alpha_{mn}\right] \frac{V(R^2)}{R^2}$$

or $$\frac{\partial V(\xi_1^2)}{\partial \xi_1^2} = \frac{\partial V(R^2)}{\partial R^2} = \text{constant}$$

Therefore,

$$V(\rho^2) = a\,\rho^2 + b \qquad \ldots \qquad (A\,9)$$

where a and b are constants.

We, thus, see that there are only two kinds of s.p. potentials whose eigenfunctions can be thrown into a product or a finite sum of products of functions of intrinsic and centre of mass coordinates. One is the harmonic oscillator potential and the other is the constant potential when a = 0 in (A 9) and whose eigenfunctions are plane waves.

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